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Using Think-Alouds to Examine Pre-Service Elementary Teachers' Visualization of Fractional Concepts

Barbara B. Leapard

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Barb Leapard's chapter discusses a problem that might cause many of us nightmares – fractions. Barb's students really need to understand how to work with fractions, because they will soon be teaching this subject to their elementary school students. The need to teach a subject requires a significantly higher form of learning than most students achieve. Barb's past experiences indicated that students often do not achieve this deep understanding – they may have a rote understanding of rules for dealing with fractions, but this will not be all that useful to them in a few months, when they are teaching inquisitive elementary school students how to work with fractions.

Barb's approach to this project was quite innovative. She used "think-alouds" to record (audio and video) her students working on fractional problems. By forcing them to be explicit about their processes, Barb was able to identify and catalog many common errors. And, by getting students to explain what they were doing as they did it, Barb's students learned fractions as if they were teaching it. This becomes a nice example of situated learning: Barb's students were learning in an identical situation to that in which they would have to apply their knowledge. Moreover, listening to and watching the tapes will provide a useful source of data for Barb to use in examining her teaching; it is safe to say that she will never teach this course quite the same way as a result of this experience.

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Rationale for Study

Eastern Michigan University is known nationally for its preparation of pre-service teachers. In particular, our elementary pre-service teachers have a reputation of being very well-prepared and are sought after throughout the country. Pre-service elementary teachers are required to take two mathematics content courses and a mathematics methods course before student teaching. The purpose of the content courses is to ensure that the pre-service teachers understand conceptually the mathematical content for kindergarten through sixth grade. The major overriding goal of the methods course is to prepare pre-service elementary teachers to teach all mathematical concepts for kindergarten through sixth grades. This research study focuses on one of the most important goals, and perhaps one of the greatest concerns, of the methods course: preparing pre-service elementary teachers to teach fractional concepts.

Having taught math methods for pre-service elementary mathematics teachers for several years, I became acutely aware of the difficulty these students encountered when explaining fractional concepts. To address my concerns, I implemented a fraction project in which the pre-service teachers were required to represent fractions using area models, consisting of diagrams with shaded regions (see Figure 5-1), and to work out the problems algorithmically. When I graded the projects, I discovered that the students were working out the problems algorithmically first, then they were making drawings based only on the answers to the problems, not using the original fractions as I intended. In addition, I found that students' representations of wholes were different sizes within a given problem.

Figure: 5-1: Sample Picture of Fractional Area Model
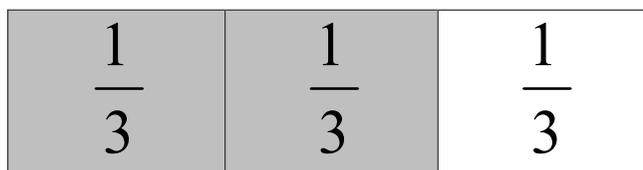
The figure represents a piece of paper divided into thirds, of which two are shaded. This would be how students represent the fraction ⅔ using an area model. Students could then physically manipulate their fraction strips to solve fractional problems in a concrete fashion.

Therefore, I added a component to the project that I felt would address these concerns. I required the students to create paper fraction strips of the original fractions in the problem, then to continue to the correct answers using appropriate modifications of the original strips. I asked the students to model the problems by starting with the concrete representation (strips), then proceeding to the semi-concrete (drawings), and finally to the abstract (algorithms), a progression advocated by Bruner's structure-oriented theory of learning (1990). However, these interventions appeared to be ineffectual, judging by the results on the fractional portion of the exam at the end of the semester. This was of particular concern to me since I knew that the pre-service teachers would not get any further feedback about their misunderstandings before they were required to teach children fractional concepts.

In order to address my concerns, I attempted another intervention to improve the pre-service teachers' understanding of fractions and their ability to teach fractional concepts. In fall 2007, I joined a cohort of Scholarship of Teaching and Learning (SOTL) participants to facilitate creating a research project that would attempt to improve on the current situation. With the input of Jeff Bernstein, Karen Busch, and others in the SOTL group, it was suggested that I use "think-alouds" in my project to gain insight into what my students were thinking as they solved fractional problems. In previous years, prior to the SOTL study, students had completed their fraction projects outside of class. I was hoping to gain insight into their thinking processes by videotaping and audio taping the students as they completed the project in class. I used the think-alouds not only as a way to help me see how my students were learning the material, but also as a device to help my students assess the learning of their students in the future. Using this innovative research method as a teaching device was certainly appropriate for a SOTL project. In particular think-alouds helped to make the pre-service teachers' learning visible since I was able to get a glimpse into their understanding of fractional concepts and into the type of teaching they would likely implement in their future classrooms.

I am not alone in my concern over pre-service elementary teachers' understanding of mathematical concepts and their ability to implement desirable pedagogical practices in mathematics. Other mathematics education researchers have documented pre-service teachers' lack of content knowledge in mathematics (Ball 1990; Ma

1999). Others have found that pre-service teachers are not always able to understand the reasoning behind their students' responses (Even and Markovitz 1995; Even and Tirosh 1995). Stipek, Gearhart and Denham (1997) found that effective implementation of a conceptually-based mathematical curriculum requires not only that teachers have a deep understanding of mathematical content, but also a deep understanding of how students build mathematical knowledge. With respect to fractions, Davis, Hunting, and Pearn stated: "The teaching and learning of fractions is not only very hard, it is, in the broader scheme of things, a dismal failure" (1993, 63). Although the researchers were referring to practicing elementary teachers, pre-service teachers become part of this group in a relatively short period of time after completing their methods courses. Indeed, most worrisome to me was knowing that the pre-service teachers in my methods classes would be teaching fractions with a rather limited understanding of the basic concepts, both in their student teaching and subsequent in-service teaching.

With these concerns in mind, one major component of this project was to determine how pre-service elementary teachers approach creating representations of fractional concepts and to determine their abilities to explain their representations. In addition, cataloging common errors that the pre-service elementary teachers make when creating representations of fractions, and the reasons why they occur, were explored as another part of the project.

These concerns are extremely important in elementary mathematics. If pre-service elementary teachers have difficulty understanding fractional concepts, it stands to reason that they may have difficulty explaining these concepts to their future students. As The National Mathematics Advisory Panel states: "It is self-evident that you cannot teach what you do not know" (2008, xxi). These misunderstandings may affect their students as they travel through the typical topics awaiting them in more advanced mathematics courses. Working with ratios, proportions, percentages, solving algebraic equations with fractions, solving complex algebraic fractions, working with probability, and finding derivatives and integrals in calculus are just a few of the topics impacted tremendously by fractional understanding. Students' lack of understanding of fractions may also cross over into many other disciplines such as biology (genetics), physics, chemistry, and the med-

ical fields.

Think-Alouds as a Vehicle for Understanding Pre-Service Elementary Teachers' Understanding of Fractional Concepts

According to Calder and Carlson, think-alouds were

> originally developed by cognitive psychologists in order to study how people solve problems. The basic idea behind a think aloud is that if a subject can be trained to think out loud while completing a defined task, then the introspection can be recorded and analyzed by researchers to determine what cognitive processes were employed to deal with the problem. …Think-alouds offer a promising method to uncover what conventional assessment methods often miss: hidden levels of student insight and/or misunderstanding (2002, 1).

Accessing hidden levels of student insight is the essence of this research project; it can be difficult in teacher-centered classroom environments to capture what students are thinking.

Carr finds think-alouds to be a useful alternative assessment in the classroom. She utilizes them in the classroom as an opportunity to "assess students' comprehension strategies, to discover how they deal with comprehension difficulties, and to integrate this information into lessons" (2002, 159). She contended that as students increasingly use think-alouds, they begin to understand their own mental processes better as they solve problems, and they discover more about themselves as learners.

Silbey is another strong advocate for the use of think-alouds in elementary math classrooms. She advocates having teachers verbalize solutions to math problems or share solution processes with students in order to give them the opportunity to "effectively crawl inside your brain" (2002, 26). Her contention is that think-alouds allow teachers to share their more sophisticated thought processes with students who can then apply this more sophisticated type of thinking in new mathematical situations.

Think-alouds were implemented as a vehicle for understanding the difficulties, if any, that pre-service mathematics teachers encounter

in explaining fractional concepts. Seeing students verbalizing as they worked out problems using the three different models (concrete, semi-concrete, and abstract) gave me insight into their conceptual difficulties in dealing with the different models. Although the students found the think-alouds awkward at first, they began to get comfortable with the procedure as they continued through the process.

One of the benefits of implementing think-alouds in this project was that the results would contribute to the research on utilizing think-alouds in mathematical situations. At present there is a paucity of research in this area. In addition, this project may prove to be useful for many other disciplines as a method of understanding students' thinking as they solve complex problems. Disseminating the results from this project may also encourage others to attempt similar projects in their disciplines. Another consideration for using think-alouds was that possibly more useful data could be gleaned from this unique qualitative approach rather than from strictly quantitative approaches.

Situated Learning as a Theoretical Framework for the Study

Collins (1988) defined situated learning as learning in which students gain knowledge and skills in the same or similar environment to the one in which they will eventually use those skills. One example of situated learning is the nursing student who learns by working with patients in a hospital rather than learning in a classroom removed from the medical environment. Another example is immersing a student who is learning a foreign language into a situation where only that foreign language is spoken.

Brown, Collins, and Duguid (1989) are usually credited with developing situated learning theory; after them, Lave and Wenger (1991) expanded on the idea of situated learning and are often associated with the theory. According to Lave and Wenger, learning takes place best in the context in which it is situated. They contrasted that to abstract, out-of-context learning that they suggested takes place in many classrooms. In addition, they suggested that situated learning has a strong social component that they referred to as a community of practice. In summary, they theorized that learning is embedded within an activity, and within a context and culture.

Because situated learning constitutes the overall theoretical

backdrop for this study, using think-alouds seemed to be a reasonable strategy. It involved the pre-service teachers learning fractional concepts in a setting much like the one in which they will subsequently use this knowledge. In addition, using think-alouds gave the pre-service teachers in the study invaluable practice with a topic that has traditionally been taught algorithmically. As noted above, verbalizing while thinking about what they were doing mathematically proved to be difficult at times for those who had not had any prior opportunities to practice that skill. However, verbalizing mathematically will be expected of the pre-service teachers in subsequent teaching situations, so the practice they garnered in this study will be of benefit to them in the future. In short, the think-aloud method helped me to teach pre-service teachers to teach mathematics by actually having them teaching mathematics. This method may help them assess their own future students' mathematical learning by making the students' understanding of mathematical concepts more visible.

Gathering Data/Methods

In winter semester 2008, as part of the SOTL seminar, I developed a research project that had several components. I captured think-alouds on videotape and audiotape in order to gather data on how pre-service teachers approached explaining fractional concepts and to determine which errors they made consistently. I developed a survey which was administered at the end of the research project to sample students' subjective responses to the think-aloud strategy. In addition, I used data from the fractional portions of the final exam to provide a quantitative measure of the pre-service teachers' learning. This amalgam of various methods allowed me to be fairly certain of my findings because of the triangulation of data gathered from each method.

Pre-service teachers from two MATH 381 classes (total $n= 38$) at Eastern Michigan University participated in videotaping and audiotaping of think-alouds as they created the concrete (paper strips), semi-concrete (drawings), and abstract (algorithmic) representations of addition, subtraction, multiplication, and division fractional problems. MATH 381 is the general elementary mathematics methods course required of all pre-service elementary education majors at East-

ern Michigan University.

The participants in the videotaping and audiotaping of think-alouds included 33 females and 5 males. Most of the participants (33) were general elementary pre-service teachers who had had only two content courses prior to the methods course. However, there were five elementary math majors or minors who had had seven to ten elementary math courses prior to the methods course. The students in the class were primarily seniors who had completed most of their coursework and were planning to student teach within the following year.

The students were invited to participate in the study after an explanation of what the study would entail. Consent forms were passed out prior to the study, and only 6 students (out of 44) chose not to participate in the videotaping and audiotaping of the think-alouds for the study. Thirty-eight students agreed to participate in the survey on think-alouds and to allow me to analyze their written work for the study.

Video cameras were set up to capture the think-alouds of two groups, and tape recorders were set up to capture the think-alouds of three groups in the classroom. For most of the videotaping, the cameras were set up at two stations in the classroom, generally at opposite sides of the classroom, and two groups of students volunteered to be videotaped, usually based on their proximity to the video cameras. The other three groups of students agreed to work with the tape recorders. The videotaping and audiotaping took place simultaneously in the classroom during two consecutive class periods. The participants worked on each problem in sequence, with the first student creating the fraction strips, followed by another student creating the drawing, and a third working out the problem using the standard algorithm. The students rotated the three tasks so that each person in the group had practice with the various representations.

The videotapes and audiotapes were then transcribed, and common errors in solving the fraction problems were noted. The constant comparative method advocated by Glaser and Strauss (1967) and Lincoln and Guba (1985) for coding qualitative data was utilized in this study. This method uses inductive category coding as data items are recorded and classified. Data items are analyzed and are placed in categories that undergo refinements as the coding continues. Through this constant comparison of data, broad categories are established.

In addition to the videotapes and audiotapes, the participants' written models (diagrams and algorithms) and physical models (fraction strips) that were created during the think-alouds were obtained and analyzed. Qualitative data were obtained from the think-alouds to determine how the pre-service teachers verbalized their thinking about fractions, to determine which common errors occurred as they worked with the fractions, and to begin to understand why those errors occurred.

A survey was given to the students to capture their opinions of using the think-aloud protocol utilized in understanding the fractional models. Five questions were given to the students at the end of the project:

- In your opinion, how effective do you think the think-aloud protocol is for teaching math concepts?

- You all worked in groups during the think-aloud process. How different do you think the process would have been if you had been working individually on the problems?

- Do you think you would consider using think-alouds when you teach children math concepts? If so, why? If not, why not?

- On this project, you progressed from the concrete (strips) to the semi-concrete (drawing) to the abstract (algorithm). What are your thoughts on using this process when learning fractional concepts?

- Do you think you would consider teaching children fractional concepts using the concrete/ semi-concrete /abstract process? Why or why not?

Students' reflections on the think-aloud protocol were analyzed as a part of the research project. In addition, results of the fractional problems on the final exam were analyzed in order to determine the number of errors that occurred and to determine if there was a pattern of errors that could be addressed in future classes.

Results of Study

Results from Videotaping and Audiotaping of the Think-Alouds

At first glance, it appeared that relatively few errors were made on the videotapes with respect to the fractional concepts. However, after transcribing the videotaping and coding the errors, there were actually 26 mathematical errors or episodes of confusion in 52 problems. A similar frequency of errors was noted on the audiotapes.

Using the constant comparison method for analyzing the data, several broad categories emerged, four related to students' understanding of mathematical concepts and two related to concerns about their ability to explain these concepts. These categories are delineated below with a typical comment that illustrates the type of error or confusion that occurred during the think-alouds. In addition, an explanation of why the error most likely occurred is included.

Using incorrect physical models. There were several instances of incorrect physical models being used during the videotaping and audiotaping. For instance, using the comparison model rather than the take away model for subtraction with the fraction strips and diagrams caused a great deal of confusion. In the comparison model for subtraction, the original parts of the problem are created and compared to each other, and the difference between the two is noted. In the take away model, the amount to be subtracted is taken away of the original amount. The take away model is always preferable when using physical models because the logic is easier to follow, a recommendation alluded to earlier in the semester when working with whole numbers. The reason why students try to use the comparison model most likely stems from the carryover from physically adding fractions, where the original parts are created next to each other before being added.

In one case in one of the videotaped segments, the pre-service teachers used the comparison method to create fraction strips for the problem $2 - 1\frac{1}{6}$ and simply could not continue. They resorted to converting both numbers into improper fractions ($\frac{2}{1} - \frac{7}{6}$) as they had been taught in elementary school. They still could not determine how to work with the problem physically. Eventually they resorted to working out the problem using the standard algorithm, which also did not help them with the physical representations of the problem. It was not until

they tried the take away model that the physical representations of the problem became clear to them.

Another problem that occurred very frequently with the models was the use of different sized wholes when creating diagrams. The purpose in using fraction strips was as a constant reminder that the wholes should be the same size in the diagrams that accompanied the problems. This was stressed repeatedly when practicing think-alouds before they were videotaped and audiotaped. However, this error still occurred frequently in diagrams for all operations during the think-alouds. When asked why the same sized wholes were not used, students stated that they did not think it would make any difference. That is, understanding what the whole is in each problem is something that they had not really thought about before, despite its utmost importance when working with fractions. This error may have had its roots in the development of fractions that the students were exposed to in elementary and middle school mathematics instruction. Perhaps they had never had to draw diagrams to explain the fractional concepts.

One of the interesting errors that occurred with the models was the use of the commutative property to change the problem into the reverse of the given problem. For example, given the problem ½ x ¾ , which means to take ½ "of" ¾ , several groups of students created the fraction strip for the ½, then they took ¾ of that, which is the representation of the problem ¾ x ½, or ¾ "of" ½. Although it is true that the mathematical result of ⅜ is the same for both versions of the problem and that the difficulty level for both representations is the same, the physical representations are completely different. This error most likely occurs because students have been told repeatedly that the commutative property gives the same answer for addition and multiplication when working with whole numbers, such as for 3 x 4 and 4 x 3. Although this is true, the physical representation of three groups of 4 objects is not the same as four groups of 3 objects.

Working from abstract to concrete models rather than from concrete to abstract models. Bruner's (1990) theory of learning, which includes the enactive (concrete), iconic (semi-concrete) and symbolic (abstract) modes, was used as a backdrop for this portion of the study because it has been shown to be an effective sequence for teaching mathematics. Ideally students were to follow this sequence when working with the various fractional representations during the think-

alouds. However, I found that in many cases, students were determining the answers using the traditional algorithm, then were creating the concrete and semi-concrete representations. Without the evidence from the videotapes and audiotapes, I may not have realized that this was the order in which the students created the representations. In many other cases, students created diagrams of the original problem, then worked out the problem algorithmically, followed by a correct diagram based on the answer they found. Unfortunately, the diagrams of the original problem and the diagram of the final answer had no connection to each other, and as before, the wholes were different sizes. This error occurred either because students had no conceptual understanding about how to make the connection between the original problem and the answer, or perhaps they hoped that I would not notice the leap from one part to the other as I graded their written work.

Comments from students who struggled with the progression from concrete to abstract representations indicate that their future teaching may mimic the way in which they were taught prior to this course, rather than the way I was encouraging them to teach in the manner of the think-alouds. Two typical comments that showed ambiguity with the concrete to abstract representations were:

> I am going to have to do it abstractly first, then do it the concrete way. Because doing it the other way around, I am lost, and I am not understanding it.

> I like doing it rotely [algorithmically]. It makes sense to me. I'll do it the new way [concretely], but I'll do it the old way to see if I have it right.

Incorrect mathematical statements. One problem revealed in this study is the difficulty that pre-service teachers had when attempting to express mathematical concepts verbally. It is vitally important that they use the correct terminology and the correct mathematical statements when they teach. Not only would they perpetuate incorrect terminology, but they also would be perpetuating misunderstandings of the properties of mathematics. Some of the errors that were noted were statements such as, "7 – 10 = 3", when subtracting numerators of fractions; "4 can go into 2, two times", when changing a fraction to low-

est terms; and "I want ¾ of that" when referring to 1½ divided by ¾. In the first two cases, the student is assuming that subtraction and division are commutative, which is incorrect. Using subtraction and division statements interchangeably can have a deleterious effect on their future students when they take algebra. In the last statement, "of" something refers to multiplication, not division. The correct meaning for this division problem is, "How many times does ¾ fit into 1½ ?"

In addition to these errors, there were several instances of mathematical errors that could be serious problems in the elementary classroom because they would skew the students' understanding of mathematical concepts. A statement that one student made, "¼ x 2 = ¼ x ²⁄₁ or ²⁄₂, which it could be, but it doesn't matter" indicates a lack of understanding about equality of numbers. (²⁄₁ and ²⁄₂ are not interchangeable.) I would conjecture that these errors occurred in all of these cases because pre-service teachers have either been exposed to them somewhere in their mathematical backgrounds, most probably in the K-12 classrooms, or they misinterpreted the statements of their mathematics teachers prior to this course.

Difficulty with division of fractions. Some of the most difficult problems that the pre-service teachers encountered centered around division of fractions, particularly with division of fractions which involved a remainder. Because many of the pre-service teachers were taught how to divide fractions algorithmically, they have not spent much time understanding, or even questioning, the "why" behind division of fractions. As one student stated, "Don't know why, but you are supposed to invert and multiply by the second fraction."

When the division of fractions involved remainders, students were unsure what to do with the leftover amount. For example, given the problem ⅜ ÷ ¼ , students were unsure how to find out how many times ¼ "fit into" ⅜. The correct way is to change ¼ into ²⁄₈ and to fit ²⁄₈ into ⅜ one whole time and ½ time. A typical chain of logic that many pre-service teachers used was the following: "¼ goes into ⅜ only once. But it has a remainder of 1. Could be 1½ or 1 with remainder of ½ or remainder of ⅛ even." It is obvious that there is considerable confusion over what should be done with the remainder. The reason that this type of error could occur is that it parallels the confusion over what to do with remainders when dividing whole numbers and decimals. Because there are several different ways to write remainders in

those situations, this may carry over into the fraction realm. However, the pre-service teacher should realize that ½ and ⅛ could not both be given as answers to the same problem.

Confusion over conceptual representations. Captured in the videotaped and audiotaped sessions were many instances of confusion over conceptual representations that may not have emerged in typical assessments. Pre-service teachers felt comfortable discussing with each other the confusion that the physical models caused them as they worked through the problems. Two typical comments that illustrate the confusion some pre-service teachers encountered when working with the conceptual representations are included below:

> The thing I was getting confused about…When you are writing it out and like when you do…like the common denominator is the bottom, why is it that we don't add the denominator in the bottom? Like, I don't even know why we don't do that, and how do you teach kids, like, you just don't add it?

> I mean, I understand ¼ of the 2 which would be ½ of the whole, but what happens to the other ¼? Does it just kind of disappear? [This statement relates to the problem ¼ x 2.]

One reason this type of confusion may occur is that often mathematical concepts are taught algorithmically, rather than conceptually, in elementary and middle schools. Although the pre-service teachers throughout the years may have been able to work out problems successfully using standard algorithms, they may have not understood the conceptual underpinnings of those problems.

Lack of confidence in ability to explain fractional concepts. Another emerging theme throughout the analysis of the think-alouds was that of a lack of confidence in the students' abilities to explain fractional concepts. In a few instances, pre-service teachers explained problems correctly, but then began to second-guess themselves and would ask the members of the group if they explained the problem correctly. In other cases, the pre-service teachers would stop the audiotaping because they weren't sure if their logic was correct. A typical comment was, "We aren't taping anything right now because we are confused." Another pre-service teacher lamented that, "I know the math, but I

don't know how to show it." Both of these comments illustrate the frustration that they felt when struggling to work out problems in a way that was uncomfortable to them. Again, the most likely reason this type of comment was made was because of their backgrounds in working with fraction problems before entering the methods course. Many students stated that they had never been shown, nor had they been required to show, representations of fractions in the manner of the think-alouds.

These categories of fraction errors and confusion are significant with respect to situated learning because the students were learning fractions in a setting that resembles that in which they will find themselves explaining fractional concepts to children. They are of concern because of their pervasive nature in building their future students' understanding of mathematics. Using incorrect models and sequencing, incorrect mathematical statements, and projecting general confusion about fractional concepts would be detrimental to their teaching and would create mathematical confusion for future generations of math students. To counteract this confusion, increasing the emphasis on mathematical content in addition to pedagogy during the methods course seems well-advised.

Students' Written Work

Students in the research project turned in the fraction strips, the drawings, and the algorithmic work that accompanied the video- and audio-taping. The errors on the students' work were minimal, since the students were able to perfect their work before turning it in. However, without the videotapes and the audiotapes, the assumption would be that the students had no errors in their thinking and that they had a very good understanding of fractional concepts. Thus, the value of think-aloud research method becomes clearer – it makes visible errors and misperceptions in student thinking that would normally have remained hidden even to a conscientious instructor who carefully examined student work.

Survey Results

Students completed a survey concerning the think-aloud process that was utilized during the research project. The results of the survey are posted in Table 5-1. Student comments were overwhelm-

Table 5-1: Results and Sample Comments from Survey of Students
(*n=28*)

Questions About Think-Aloud Process	Representative Student Comments
1. In your opinion, how effective do you think the think-aloud protocol is for teaching math concepts? (Positive comments: 22; Negative: 2; Mixed: 4)	• *Positive:* "I thought it was helpful as future teachers because it made me realize how difficult it can be to explain such concepts." • *Negative:* "I think that it is effective for some people, but difficult for me because I cannot verbalize my thinking." • *Mixed:* "It was difficult to put into words all of my thought processes. Sometimes I could not adequately explain what I was actually doing with the strips. However verbalizing did help me understand more."
2. You all worked in groups during the think-aloud process. How different do you think the process would have been if you had been working individually on problems? (Positive comments: 28)	• *Typical positive comment:* "I don't think it would have been as effective. Working in groups gave me the chance to help others. It was a great opportunity to put my thoughts into words."
3. Do you think you would consider using think-alouds when you teach children math concepts? If so, why? If not, why not? (Positive comments: 27; Mixed: 1)	• *Typical positive comment:* "Yes, especially in small groups. This helps students teach each other and also helps teachers understand how their students think!"
4. On this project, you progressed from the concrete (strips) to the semi-concrete (drawings) to the abstract (algorithms). What are your thoughts on using this process when learning fractional concepts? (Positive comments: 19; Negative: 5; Mixed: 4)	• *Positive:* "I really liked using this process. I now finally understand fractions better than I ever have!" • *Negative:* "This was a bit difficult. I found myself doing it abstractly first (the way I was taught). This helped me in doing the concrete part because it was a bit confusing for me even though I am a visual person." • *Mixed:* "I thought the concrete portion was the most difficult and semi-concrete second difficult. As time passed, I began to understand the concept a lot better and the task became easier. It turned out to be a lot of fun."
5. Do you think you would consider teaching children fractional concepts using the concrete/semi-concrete/abstract process? Why or why not? (Positive comments: 25; Mixed: 3)	• *Positive:* "Yes, because I think having a tangible, visual object really helps students understand fractions better and know exactly what a fraction is rather than just thinking about it in the abstract first." • *Mixed:* "I would consider the process, but it may be a source for more confusion.

ingly positive about using the think-aloud protocol, using the concrete/ semi-concrete/ abstract process of representing fractions, and working in groups as they worked through these representations.

The survey reveals how think-alouds were effective as a research tool. Students agreed overall that think-alouds provided invaluable practice for teaching fractional concepts. Although they were not aware that situated learning was the backbone of this research experience, they clearly felt that this experience involved them in learning fraction concepts in a setting much like the one in which they will subsequently use this knowledge. This gives further credence to the theoretical underpinning of situated learning for this research..

Although students were sometimes reluctant or unable to follow the concrete, semi-concrete, abstract sequence at times during the study, there was a general consensus that this sequence was something to strive for whenever possible. In addition, the students found working in groups to be a positive experience because it allowed them to discuss any problems they may have arisen during the think-alouds.

Final exam problems. The fractional problems on the final exam that closely matched those in the research study were analyzed for errors. The actual problems and the percentage of students who made errors on the problems *(n = 34)* can be found in Table 5-2. The discussion that follows shows several of the types of errors that were made for each problem.

Table 5-2: Results of Fractional Portion of Final Exam *(n=34)*

Question	Fraction Problem	Number of Students with Incorrect Answers
A	$\frac{5}{6} + \frac{1}{2}$	$\frac{6}{34}$ or 17.6%
B	$1\frac{3}{4} - \frac{7}{8}$	$\frac{13}{34}$ or 38.2%
C	$\frac{3}{4} \times \frac{4}{5}$	$\frac{7}{34}$ or 20.6%
D	$\frac{1}{6} \times 2$	$\frac{15}{34}$ or 44.1%
E	$\frac{7}{8} \div \frac{1}{4}$	$\frac{18}{34}$ or 52.9%

Problem A: $\frac{5}{6} + \frac{1}{2}$

For this problem, there were many similarities to the errors found in the videotaping and audiotaping: work was done algorithmically first, then drawings were made to match; the original parts of the problem

were drawn and the answer was drawn without any connection to the original parts of the problem. However, this problem had the fewest number of errors (6/34 or 17.6%). The correct answer for this problem is 1⅓, obtained by changing ½ to ³⁄₆, then adding ⁵⁄₆ and ³⁄₆ to get ⁸⁄₆ . Changing the fractions to lowest terms yields 1²⁄₆ or 1⅓.

Problem B: 1¾ - ⅞
As before, the original fractions were drawn, and students stopped at that point, not sure how to proceed. Another error that occurred on this problem was that the drawings were inconsistent with the original problem. One interesting example of this was a diagram that showed confusion over the whole. A diagram that represented 1¾ was created, with the whole divided into fourths with all the fourths shaded in, and a second whole divided into fourths with three of the fourths shaded in. However, the diagram was labeled as ⅞. The pre-service teacher reasoned incorrectly that there were 8 total sections with seven of the sections shaded in. One correct way to sketch a diagram of this problem is to take two strips, shade in 1¾, then change the ¾ into ⁶⁄₈. The other whole should be broken into eighths as well. This results in ¹⁴⁄₈. Then ⅞ should be taken away from the ¹⁴⁄₈ to yield the leftover amount of ⅞.

Problem C: ¾ x ⅘
Many students drew the original two fractions using fraction diagrams, but were unable to continue. Several students drew the original two fractions, then they simply created a diagram of the answer that they found by working out the problem algorithmically, similar to the patterns found on the videotapes and audiotapes. The correct answer for this problem is ⅗, obtained by canceling out the 4's and multiplying numerators and denominators to get ⅗.

Problem D: ⅙ x 2
Many students drew 2 wholes, shaded in the entire area and stopped at that point because they didn't know how to proceed. Several continued on to a next step by shading in ⅙ on one of the wholes and then stopped at that point. In addition, similar to the findings in the videotapes and audiotapes, many students worked out the problem algorithmically, then they just shaded in the answer ²⁄₆. One correct way

to create the diagram for this problem is to make two wholes, then to divide both wholes into sixths. Then by using the distributive property [⅙ x 2 = ⅙ (1 + 1) = ⅙ x 1 + ⅙ x 1 = ⅙ + ⅙ = ²⁄₆], ⅙ of each whole would be shaded in to obtain ²⁄₆.

Problem E: ⅞ divided by ¼
Just as in the videotapes and audiotapes, the division of fractions problem produced the most errors. A persistent error that occurred was working out the problem algorithmically first, then shading in the final answer. On the other hand, several students shaded in the ⅞ and the ¼ correctly. They proceeded to change the ¼ to ²⁄₈ . They then fit the ²⁄₈ into the ⅞ three times all of which is correct to this point. However, the prevalent error here was that the students did not know what to do with the leftover piece. The correct answer should have been 3½ , since the ²⁄₈ fit into the ⅞ three whole times and half of another whole time. However, many students obtained the answer 3⅛ since ⅛ was left over. This error is also parallel to the remainder errors obtained on the videotapes and audiotapes.

Although the primary goal of the methods course is to prepare pre-service teachers to teach mathematical concepts for kindergarten through sixth grade, the results of the fractional portion of the final exam show fairly clearly that a secondary (and prerequisite) goal should be to make sure that the pre-service teachers understand fractional concepts. It would be difficult for pre-service teachers to teach fractional concepts for understanding without a thorough understanding of those concepts themselves.

Discussion

Videotaping was an excellent adjunct to the students' drawings and algorithmic work on each problem. Looking solely at the written work would have suggested that the students had no difficulty solving the fractional problems. However, when watching the students on the videotapes, I could see the confusion and incorrect logic that occurred fairly frequently. One of the negatives of using videotaping in the classroom was that many students felt self-conscious at first. Some of my students expressed a reluctance to be videotaped because they feared they might make a mistake. Other students, however, were very confi-

dent in front of the cameras and volunteered for that option. Interestingly, some students seemed even more confident in themselves when the cameras were capturing their work than they did at any other time during the course. There was virtually no hesitation and no pauses between representations. In essence, they looked like they knew exactly what they were doing. An interesting occurrence that happened during the videotaping was that the students did not discuss the problems with each other as would normally happen when working in a group. Each one performed his part without any input from the other students. It was unclear if they thought they were not allowed to discuss the problem because of the videotaping, or if they simply did not need any help in finding the answers.

Making audiotapes seemed to be the preferred option for most of my students during the research study. Perhaps because they felt more control to turn the machine off if they thought they were making mistakes, students were more at ease with using audio over video. Unfortunately, some of the best discussions were lost because students wanted to perfect their problems before turning on the tape recorder. In several instances I had to remind the students that I was trying to capture their thinking and discussion as they thought through the problems for the first time. Although the students making the videotapes tended not to discuss the problems with each other, the students making audiotapes discussed the problems before, during, and after the audiotapes were running. It appeared to be a more relaxed atmosphere for those students compared to those being videotaped.

The survey results concerning the think-aloud protocol, the concrete to semi-concrete to abstract process to explain fractions, and group work during the think-aloud protocol were overwhelmingly positive. The pre-service teachers found that verbalizing their thought processes as they used the think-aloud protocol was awkward at first. However, as they realized how closely the think-aloud protocol approaches what they will be doing when they are teaching, they began to see the value in it. Although some students did not appreciate the concrete to abstract process personally, they agreed that it was probably the best way to teach mathematical concepts. All students were very definite about working in groups as a positive situation. They felt that if they had had to work out the problems individually that they would not have understood the concepts as well. Interestingly, even

those who participated in the videotaping shared this view, although they did not appear to discuss the problems with each other in their groups.

One of the most beneficial aspects of this research project for me was gaining insight into the errors that students made concerning fractional concepts. Although I had some indications in the past of the types of errors that students make, I had never officially kept track of those errors. For example, I knew that many previous pre-service teachers had used different-sized wholes in the same problem when drawing their diagrams, which is why I had them use concrete fraction strips as a reminder for that concept. However, I was surprised at the number of subtle and not so subtle errors that occurred when they were explaining their problems verbally. Moreover, the confusion and lack of confidence that seemed to pervade their explanations were troublesome. As I pointed out to them, teachers must have both content and pedagogical knowledge in order to teach well. Otherwise their future students will lack confidence in them as teachers. Because of this research project and knowing now the consistent errors that are likely to be made, I plan to utilize the errors to direct my teaching.

One of the purposes in this SOTL project is to have us step back and look at our teaching practices and to determine how it relates to our students' learning. Having given this matter a great deal of thought, I plan to implement the following changes when I teach the methods course in the future. In order to encourage correct model usage, I plan to use several different models in addition to the fraction strips. Fraction circles, fraction squares, and fraction bars are among some of the models that are available commercially for representations of fractional concepts, and they would provide the pre-service teachers with a deeper understanding of the importance of the whole in a given problem.

In addition, I plan to have students create both physical representations of commutative statements such as ½ x ¾ and ¾ x ½ in order to have them discover the subtle differences between these equivalent statements. I will also stress that subtraction and division are not commutative and that using the statements interchangeably could have negative effects on their future students. As far as following the sequence of concrete to abstract models, I will continue to stress that that sequence is the preferred one in most of the research-based

literature on learning mathematics, and I will stress the connections between the various representations. In addition, I will recommend that the pre-service teachers use this sequence when working with children on their major teaching projects for the semester.

Overall, I plan to give students more extensive practice in representing fractional concepts, both in class using think-alouds as well as outside of class. This could help with incorrect models and with the division problems, particularly with respect to the confusion about how to handle the remainder. As far as the general confusion about explaining fractional concepts, increasing the time spent in class on think-alouds may address this problem. One other possibility for improving on the current state of affairs is to give students examples of the prevalent errors that have been made in past classes and have them determine and discuss the errors and the implications of these errors in their future classrooms. Finally, I plan to make my courses more learner-centered and to give students more time to assimilate concepts and to solve problems without quick interventions from me. Allowing pre-service teachers to create fractional representations for each other and with each other's support should improve on their understanding of fractions and their ability to verbalize their understandings.

I had hoped to see improvement in the fractional portion of the final exam at the end of the semester, so I was surprised to see the large number of errors that still persisted after implementing the think-aloud protocol in my two classes. While the videotapes and audiotapes revealed errors, they did not seem to be as serious as the conceptual errors made on the final exam. However, on closer examination, I found that students errors paralleled the errors they had made during the think-alouds.

Although the fractional errors did not decrease markedly after utilizing the think-aloud protocol in this study, I still feel that it was a beneficial exercise. Ultimately it showed the pre-service teachers that it is difficult to verbalize thoughts about a topic that they do not understand really well, and that they should begin to take steps to improve their content knowledge if they have deficits in their mathematical backgrounds.

Limitations of this Study

There were several limitations to this study. Some pre-service teachers may have already known how to determine the answers to the given fraction problems prior to the study; therefore the think-aloud protocol may have had no effect on their understanding of the fractional concepts. Working in groups may have masked errors that could have occurred if the problems had been worked out individually, and interviews with individuals may have revealed richer data. Different fraction problems may have yielded different results, and the small sample of thirteen problems used in the think-alouds may have overlooked other types of errors that students usually make when working with fractions. The videotaping and audiotaping may have created an artificial environment that caused students to react in a manner inconsistent with their normal behavior. Finally, they may have written what they thought I wanted to hear when responding to the survey about the project.

In further iterations of this research study, I would address these limitations by giving the pre-service teachers pre-tests to determine their prior knowledge of fractional concepts. In addition, I would add an interviewing component to the study in order to capture a deeper understanding of their errors and confusion. I would ask colleagues to advise me on the number and choices of problems for the research study. Although I would continue to use video cameras and tape recorders, I would give the students more practice with the think-alouds prior to the data gathering.

Conclusions

Gathering data for this project was an excellent opportunity for me to deeply analyze my teaching and my students' learning with respect to fractional concepts. I knew that students in the past had had great difficulty with fractions on the final exam, but I had not had the chance to capture the essence of their errors and the confusion that ensued as they tried to explain their thinking. Being able to videotape and audiotape the students as they "thought-aloud" was a gift to me from the students who agreed to be part of the research study. To be able to glimpse into the thinking of almost all of the students in the

class is usually just not possible when observing classes. Listening to the struggles that they had as they explained their parts of the problem has given me great insight into how I can approach teaching fractions in my future methods classes.

The situated learning opportunity that the think-alouds created was an excellent precursor to student teaching and in-service teaching for the pre-service teachers. Many of the pre-service teachers realized that verbalizing mathematical concepts, particularly those concerning fractions, was not as easy as they thought it would be. Several also indicated that they were grateful for the opportunity to practice those skills before student teaching.

Introducing the pre-service teachers in this study to the think-aloud protocol, together with the concrete, semi-concrete and abstract processes advocated by Bruner (1990), will hopefully encourage them to use these methods to model fractional concepts in their own classrooms. Knowing only the algorithms for determining the answers to fractional problems is no longer sufficient as a teaching strategy, since many of their teaching situations will require an understanding of concrete manipulatives and diagrams for those problems. In addition they should be able to explain why the algorithms work the way they do for their future students who are not satisfied with learning seemingly nonsensical rules.

Not only have I learned a tremendous amount about my students' future teaching abilities in this research study, but I have also learned a few things about my own abilities as a teacher. I found that I have a tendency to explain things too quickly and assume that students understand what I have said if they do not say anything. Several times on the audiotapes, I explained to students how to work out problems. They politely listened to my explanations, but it was clear by their next conversations that they really did not understand at all what I had attempted to explain to them. Although I thought I had done an excellent job of explaining how to do the problem, I realized that I needed to slow down to let them absorb what I was saying, or better yet, I needed to let them talk through the problem on their own until they discovered the answer. In addition, I found that not every student finds visually looking at math concepts helpful. Indeed, many students actually found the drawings more confusing than helpful. Because I am a visual learner, I assumed incorrectly that even those who are not

visual learners would benefit from making diagrams of fractions.

I consider it a privilege to have been part of the SOTL research group during the past year because it provided me with the unique opportunity to analyze my teaching and my students' learning of fractional representations. As a teacher educator, I am always looking for new interventions for teaching mathematical concepts. I was not familiar with the think-aloud protocol as a formal teaching strategy before starting this research project. However, I have found it to be a powerful strategy that I plan to utilize in my methods courses in the future. I hope that other teacher educators will be encouraged by the results of this study to incorporate think-alouds in their courses. The benefits of think-alouds include being able to listen to pre-service teachers' explanations and to provide them feedback on their use of terminology, the logical progression of their explanations, their confidence levels, and their mathematical understanding – essentially all facets of teaching mathematics rolled up into one neat package. One of the issues in any SOTL project is the potential of the ideas and results of the project to inform the efforts of others engaged in SOTL or in teaching in general. Hopefully this research project fits this purpose.

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