

10-23-2009

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Recommended Citation

Han, Xiaoxu (2008) "Teaching Elementary Linear Algebra Using Matlab: An Initial Investigation," *The Scholarship of Teaching and Learning at EMU*: Vol. 2, Article 9.

Available at: <http://commons.emich.edu/sotl/vol2/iss1/9>

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TEACHING ELEMENTARY LINEAR ALGEBRA USING MATLAB™: AN INITIAL INVESTIGATION

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Xiaoxu Han teaches linear algebra. In his quest to innovate his class, he has explored using a computer software package to do the calculations for students and remove some of the tedious algebra and arithmetic that often lead to minor math errors when working with matrices. Moreover, the software packages become necessary when matrices become too large to be solved by hand. Xiaoxu's project involved incorporating MATLAB™, his software of choice, into his course and examining the students' perceptions of how much using this program helped them to learn the course material. His first-stage analysis focused largely on student reports of their satisfaction with this curricular innovation.

Xiaoxu discovered that, by and large, his students were pleased with MATLAB™. However, upon more closely examining the data, he found that his stronger students (those who earned A's in the class) tended to like this program less than did students who earned lower grades. Xiaoxu uses the students' own words to explore the determinants of their satisfaction (or dissatisfaction) with MATLAB™ and from there attempts to explore the grade-based disparity. This discussion leads him to some interesting ideas about next stages in his teaching this course and, hopefully, in continuing this scholarly project.

Why This Project?

Elementary linear algebra is a fundamental class for all mathematics, math education, statistics, actuarial science, computer science and engineering majors. It covers the basic theory and applications of linear algebra and plays an essential role in further science and engineering coursework (Anton and Rorres 2005; Lay 2006; Spence, Insel and Friedberg 2008). The two, more advanced, upper-level linear algebra courses (Math 416 and Math 518) taught at EMU, use a purely paper-pencil approach, while the elementary linear algebra course has been a traditional paper-pencil plus graphical calculator course. This traditional teaching approach is valuable in teaching the basic concepts of elementary linear algebra. However, the classical teaching approach that we use presents challenges in that students sometimes have difficulty learning how to solve real-world application problems. Such problems involve relatively large matrices, instead of the general 2×2 or 3×3 matrices they would learn in a traditional class. These larger matrices are not solvable using paper and pencil approaches.

Integrating user-friendly computing software in teaching elementary linear algebra is one promising approach to solving this problem. With this possibility in mind, I decided to integrate Matlab™ into my elementary linear algebra class. Many different packages are available for teaching linear algebra, including Maple, Mathematica, Matlab™ and even Java applets (Kalman 1999; Kalman and Day 2001; Meel and Hern 2005; PCMI Undergraduate Faculty Program 1998).

I chose to use Matlab™ over the other programs for three reasons. First, Matlab™ is one of the most popular software programs used in science and engineering teaching across the United States. It has been widely used in teaching linear algebra, computer science, calculus, and engineering courses since the 1990s (Dunn and Harman 2002; Dunn 2004; Leon, Herman and Faulkenberry 2003; Marriott 2002). Compared with other software/technologies, it has broader applications and audiences. Second, Matlab™ has complete and efficient linear algebra computing modules. It provides interactive calculation results combined with powerful visualization functionalities. Thus, it is suitable for the exploration of relatively large matrix computing problems. Finally, the Mathworks Company, the developer of Matlab™ software, provides an affordable instruction version license to support college

teaching.

Thus, in selecting Matlab™, I aimed to expose my students to real-world problem-solving by applying basic linear algebra skills. Furthermore, from a scholarship of teaching and learning perspective, I was interested in investigating the strengths and challenges of using Matlab™ in teaching this lower level undergraduate course, in order to aid me in future course design. By exploring the experiences of my students, and working with them to try and make their learning more visible, I aimed to learn how to help students succeed in this class.

Project Implementations

There were a total of 23 students in my elementary linear algebra class at Eastern Michigan University. The 23 students (5 females and 18 males) were majoring in computer science (2), mathematics and elementary or secondary math education (12), physics (2), engineering (5), and history (1 – this student was a math minor). They included 2 seniors, 10 juniors, and 11 sophomores. None of the students had learned Matlab™ in prior coursework. However, two senior students had taken the elementary linear algebra course before and were repeating it for credit. All data discussed in this chapter were collected during the winter semester of 2008 with permission from all students in the class.

I integrated Matlab™ in the course by asking students to complete three Matlab™-based projects and some homework assignment problems. There were an additional seven general paper and pencil assignments. Students were also strongly encouraged to use Matlab™ commands to verify results of paper and pencil homework problems.

For convenience, I decided to write my own 25-page Matlab™ tutorial manual, Matlab™ “Basics for Elementary Linear Algebra,” instead of using available materials. This was mainly because almost all Matlab™ tutorial materials assumed readers had a basic knowledge of linear algebra (e.g., Etter, Kuncicky and Moore 2005; Lay 2006). The Matlab™ tutorial materials were covered in three separate lab lectures that I did during the semester.

I designed the three Matlab™ projects for students to tackle linear algebra problems involving large matrices by using the software. This is because I believe solving large linear algebra problems is one

of the most important learning goals in a linear algebra course. The difficulty levels of these projects increased gradually. These projects covered the most important basic linear algebra knowledge and applications; they focused on applying knowledge of linear algebra to solve problems that were not easily solvable using the traditional paper, pencil and calculator approach.

The specific assignments were as follows:

Project 1: Applying Gaussian elimination and inverse computing methods to solve 20×20 linear systems.

Project 2: Applying Matlab™ skills to solve linear systems using Cramer's rule.

Project 3: Applying the singular value decomposition (SVD) method in the image compression.

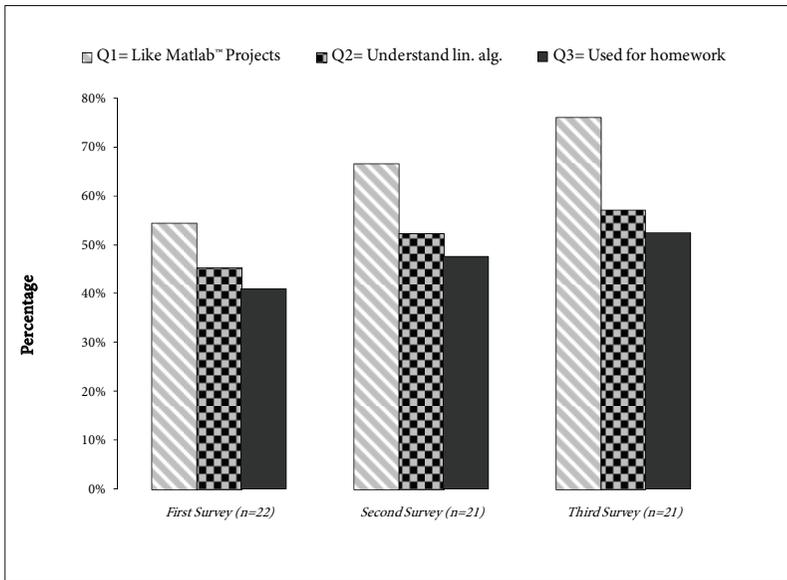
For each project, I provided students with detailed examples about how to use Matlab™ to solve similar small-scale problems. Students would go through these examples and the corresponding instructions to solve their projects. As one example, the Appendix to this chapter shows the example I gave students about how to solve a 3×3 linear system by applying Cramer's rule in Project #2.

These projects were done in groups of two to three students. The students worked together and handed in one project report as a group. Students were expected to work individually at first and then to meet together to do the final group project report. They were strongly discouraged from splitting the project into separate parts and having each student do a piece; I required each student to get involved in solving the entire problem.

Gathering Data

To collect feedback from students, I conducted a brief survey after each project was done. Students were told that they could freely express their opinions on the survey questions without worrying about their grades, as I did not review the surveys until after I had assigned final grades. Each survey consisted of three YES/NO questions. I also wanted students to tell me in the survey why they chose yes or no for each question. I only included these 'binary questions' in the survey

Figure 7-1: Comparison of Approval Rates for Survey Questions

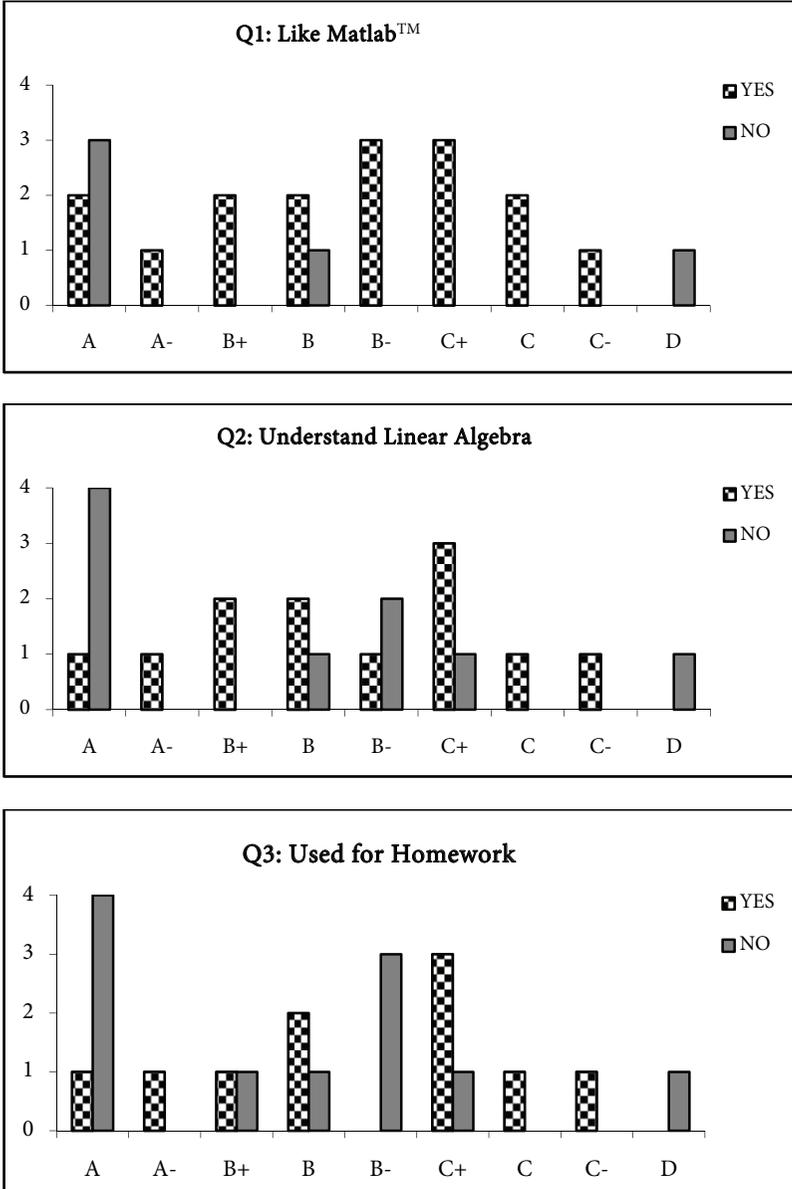


because I wanted to make the statistical analysis simple and collect all the ‘unfiltered’ answers about Matlab™ usage. For each question, I followed up the YES/NO item with a question asking why the students felt that way. The general three questions were as follows:

- Q1. Do you like the Matlab™ project(s) we did? 1. YES 2. NO Why?
- Q2. Do you think using Matlab™ will help you understand the linear algebra materials better? 1. YES 2. NO Why?
- Q3. Do you like to use Matlab™ to do your homework? 1. YES 2. NO Why?

In the third and final survey, I added one last question: Write your suggestions about Matlab™ usage in the linear algebra course for your instructor. The first, second and third surveys were conducted for 22, 21, and 21 undergraduate students, respectively, during the winter 2008 semester. The surveys were done during the 5th, 10th, and final weeks of the semester.

Figure 7-2: Student Responses from the Third Survey and Corresponding Grades. (n=21)



Analysis of Results

Figure 7-1 compares the percentage of students answering “yes” for the three questions in the three surveys. It is interesting to see that the positive feedback from the surveys increased gradually. This was one of the instructor’s initial expectations, because students would have had more chances to learn the power of this software. Obviously, students did like the Matlab™ projects they did in the class, which were ‘windows’ for them to learn and practice linear algebra materials using Matlab™.

Furthermore, the survey results indicated that more and more students thought Matlab™ helped their learning. However, at the end, nearly half the students still did not like using Matlab™ to do homework assignments. This is understandable for two reasons. First, Matlab™ software cannot handle any proof problems in the homework assignments. Second, most homework assignments consisted of at most 4×4 matrix calculation problems, which are easy to do with the traditional paper, pencil and calculator approach.

I believe it is important to pay the most attention to students’ feedback primarily from the last survey (Figure 7-2) because it provided the final opinions from students. This figure sorts the number of student responses into the three Y/N questions and provides a detailed distribution for the three questions according to students’ final grades.

For the first question (Did you like Matlab™ projects(s)?) 50% (3/6) of ‘A’ level students answered ‘YES’ and 50% (3/6) answered ‘NO’; 88% (7/8) of ‘B’ level students answered ‘YES’ and 12% (1/8) of ‘B’ level students answered ‘NO.’ 86% (6/7) of ‘C’ and ‘D’ level students answered ‘YES’; 14% (1/7) answered ‘NO.’ For the second question (Do you think using Matlab™ will help you understand the linear algebra materials better?) 33% (2/6) of ‘A’ level students selected ‘YES’ and 67% (4/6) selected ‘NO’; 63% (5/8) of ‘B’ level students selected ‘YES’ and 37% (3/8) selected ‘NO’; 71% (5/7) of ‘C’ or ‘D’ level students selected ‘YES’ and 29% (2/7) selected ‘NO.’ For the third question (Do you like to use Matlab™ to do your homework?) the ‘YES/NO’ ratios are 33% (2/6) versus 67% (4/6) for ‘A’ level students; 37% (3/8) versus 63% (5/8) for ‘B’ level students; and 71% (5/7) versus 29% (2/7) for ‘C’ or ‘D’ level students (see Figure 7-2).

From the survey results, it is interesting to see that 'B', 'C' and 'D' level students seem to feel more positively toward Matlab™ as a tool for learning linear algebra learning than did 'A' level students. Half of the 'A' level students did not like the Matlab™ projects done in the class and two-thirds of them did not think using Matlab™ helped them understand course materials. These negative responses were much higher than those observed for non-'A' level students.

This result came as a surprise to me because, when I started this project, I thought 'A' level students would welcome Matlab™ more than other students in the class. The surprising result made me recall some direct feedback I obtained in the computer lab when I was teaching Matlab™ to students. One 'A' student told me: 'I am not good at using software to do Math.' On the other hand, one 'C' student hailed using Matlab™ in the learning: 'This software is great! Can we use this to do our final exam?' Taken together with the survey results, these comments from students about their own learning have made their learning experiences in the course quite visible to me. This has helped me as I reflect on how I will teach this course in the future.

More generally, the following represent typical responses to the open-ended question about why they thought Matlab™ would contribute to their linear algebra learning.

Matlab™ is a program that is very useful to solve large problems.

It allows you to see the correct answers to problems without having to work them out by hand.

It helps in understanding linear algebra without getting lost in the algebra details.

It makes problem-solving easier and faster.

Student comments identified the most impressive part of using this software for learning linear algebra: Matlab™ is good at solving large problems in an easy, fast way. It frees students from tedious, detailed, algebra work. This was one of major goals of mine in introducing this software into the class.

Students also pointed out the disadvantages of using Matlab™ in the linear algebra learning. The following are typical comments

from students:

I did not like the program we're using. In general, I am not good with computers.

It doesn't show what it is doing, just does it for you.

It spits out answers but does not show processes or give explanations.

In addition to the reason of simply being 'not good at computers,' students did not like Matlab™ mainly because the program just does the calculations for them instead of giving explanations. This was understandable because students were just beginners at Matlab™. They mainly depended on built-in commands in Matlab™ to solve problems. Matlab™ can also show the detailed steps in problem solving; however, performing these functions requires students to have more proficiency programming with Matlab™ than they could gain from just this one class.

Although the survey results showed that students did have positive responses about using Matlab™ in their learning, there were still nearly 43% (9/21) of the students who did not think that using Matlab™ would help their learning. Among those who did not believe using Matlab™ would contribute to their learning, about 44% were 'A' level students; 33% were 'B' level students and 22% were 'C' and 'D' level students. Perhaps the 'A' level students understood course materials relatively better and faster than other students. They could also do very good algebra work without using Matlab™ to check their answers. On the other hand, 'C' level students liked using Matlab™ to check their answers and to avoid the complicated algebra work in the course.

As a final piece of the analysis, students were also asked for their suggestions about how I could best use Matlab™ in teaching linear algebra in the future. While some students' comments were along the lines of 'No Matlab™', some more constructive comments suggested that it would be helpful to provide more examples for the homework problems, and to integrate the teaching of Matlab™ with their graphical calculators. These were helpful suggestions for how to modify the way Matlab™ is used.

Discussion and Future Work

This project investigated integrating Matlab™ software into elementary algebra teaching, so that students could learn how to use this software to solve relatively large problems. I am proud to see that 76% (16/21) of the students in the class enjoyed the Matlab™-based course projects I designed. More than 57% (12/21) of the students in the class thought using this software had helped their learning.

In the future, I will continue to investigate all kinds of approaches to demonstrate to students that using Matlab™ can help their learning. For instance, I will incorporate some more challenging programming work to attract and retain top-level students' attention; this way, they would have more opportunity to explore in Matlab™. I will also put more energy into designing problems and examples for the homework assignments, to encourage more students to use Matlab™ more frequently and effectively. For this pilot, I focused on using Matlab™ to do course projects. The Matlab™ tutorial materials were also project-based and did not involve the homework assignments. Homework assignments would need to be completed by the individual students, which might make it more difficult to incorporate Matlab™; the three projects used in the course were team-based. However, while challenging, there may be much to gain from incorporating Matlab™ into the homework assignments as well as the projects.

Last but not least, I used David Lay's (2006) book, *Linear Algebra and its Applications*, 3rd edition, as the textbook this time. This book integrated Matlab™ in linear algebra teaching at a relatively high level; this book was not easy for students to read. I will use Spence's linear algebra book (Spence, Insel and Friedberg 2008) as my textbook next time. This book provides numerous step-by-step Matlab™ learning examples and it is relatively easier for students to read.

I am looking forward to teaching the elementary linear algebra course again and continuing to integrate Matlab™ software. The results of this project give me reason to believe this approach is worth pursuing again albeit with the modifications suggested by the data I have analyzed in this paper.

Appendix

Computer programming code needed for class project #2 (solving a 3x3 linear system by applying Cramer's rule):

```
clear;

A=[2 -2 4; 1 -3 1; 3 7 5]
b=[0; -5 ; 7]

column_number=size(A,2)

for i=1:column_number
    A_i=A;
    A_i(:,i)=b;
    x(i)=det(A_i)/det(A);
end

disp('This is the solution');
x'

save mywork.mat;
```

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