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An application of input-output analysis to pollution

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Abstract

Using data on pollution emissions from over 20,000 facilities in the United States, we find most pollution is released by a small number of firms, and that there is no correlation between the amount of pollution released and socio-economic indicators such as income. We apply Leontief's method of input-output analysis to determine pollution generated both by demand for final goods and inter-sector demand for intermediary goods. We find the sector that generated the most pollution in 2002, both in production of final goods and in use of intermediary goods, was primary nonferrous metal products. The analysis of several sectors, most notable motor vehicles, differs greatly when production of intermediary goods is considered.

Contents

Introduction

If certain stringent conditions are met, freely functioning markets are capable of effecting an efficient use of productive resources. However, if any of these conditions are not met, a type of market failure is likely to occur, leading to an inefficient use of resources. One such condition is that there be no externalities: the benefits and costs of production and consumption should be reflected in the prices of goods.

Unfortunately, there are many instances in which externalities occur. Externalities can be positive, if benefits from consumption or production aren't captured in prices, or negative if the costs of consumption or production are not captured. For instance, those who purchase flu shots benefit those around them at no charge, while smokers inflict costs on others without being penalized.

A significant externality in production is pollution. Many productive processes generate by-products that are released into the environment, and these by-products can have deleterious effects on people and productive resources. However, polluting firms typically are not responsible for the full costs of these effects; were they to be responsible for these costs, a more efficient use of resources would result.

Where externalities exist, government intervention is justified. Such intervention might include taxes on production or consumption of goods that generate pollution, or a subsidy for alternative products. Such policies provide economic incentives for consumers and producers to shift towards behaviors that generate less pollution. However, government actors have limited information with which to guide their intervention. For instance, it is difficult to determine the value of costs incurred as a result of pollution. It is therefore difficult for government to effect an efficient use of productive resources. If government knew precisely the costs of pollution from a firm, that firm could be taxed and the externality would be internalized. Moreover, the effects of pollution may be unevenly distributed. Thus, the government has not only to know the costs of pollution, but who is bearing those costs, so that those most affected can be compensated. Is it fair for the government coffers to be filled while others suffer the health impact from pollutants?

The void created by imperfect government intervention can and sometimes is filled by consumers. For instance, some consumers have chosen to purchase hybrid cars, and energy efficient light bulbs. However, like government actors, consumers act with imperfect information. For instance, it's possible that the productive process for nickel metal hydride batteries used in hybrid cars generates enough pollution to outweigh the benefits of their use. Moreover, the benefit to any one person from changing his or her consumption patterns to reduce pollution is likely to be minimal, thus weakening the incentive for individuals to change their behavior.

Properly considered, government actors and consumers shouldn't just consider the pollution generated in the production of final goods. They should also consider the pollution generated by the production of intermediate goods. The production of any product requires inputs from many industries. Our demand for automobiles generates pollution not just from the auto industry, but from the myriad of suppliers to the auto industry as well.

Leontief's input-output analysis was developed to analyze the total effect of changes in demand, incorporating changes in both production of final goods and changes in production of intermediary goods. It is thus ideally suited for an analysis of the total pollution generated from an exogenously determined demand. In this paper, we shall first review the theory of externalities; section 2.1 introduces input-output analysis, and section 2.2 discusses its application to pollution; section 3 covers our analysis of 2002 data for more than 20,000 facilities in the US and 195 different pollutants;in section 3.3 we consider implications for current policy initiatives in light of our results.

1 Theory of Externalities

In order to consider the theory of externalities, we must consider what is meant by economic efficiency, the measure by which economists judge an outcome.

1.1 Economic Efficiency

An outcome is considered efficient if it maximizes the net benefit to society. The net benefit to society is the total benefit less the total costs.

All activities incur some opportunity costs, the costs of foregoing alternative activities. For instance, if we devote resources to producing an automobile, the time, labor, steel, fabric and other materials cannot simultaneously be used to produce a building. In producing the automobile, we have lost every other alternative use of those resources. The opportunity cost is defined to be the cost of the most valuable alternative foregone. As any productive process is extended, the total cost incurred increases - more and more resources are prevented from flowing to alternative uses.

Additionally, the marginal costs, or costs per additional unit of production, increase. This is because of the law of diminishing returns: as any activity is extended, it becomes increasing difficult to further pursue that activity. For example, the more steel we use in producing automobiles, the more difficult it becomes to mine for the iron used in producing steel. The readily accessible iron will be used first, and additional iron will be increasingly more difficult to obtain. Therefore, total costs increase, and at an increasing rate as production is extended.

As consumption is extended, total benefit, which can be thought of as utility or satisfaction, does typically increase. However, the marginal benefits, i.e., the utility from consuming an additional unit, tend to decrease relative to the prior unit as consumption is extended. For any good, a consumer can reach a point where an additional unit of that good doesn't yield very much utility. For instance, if I were

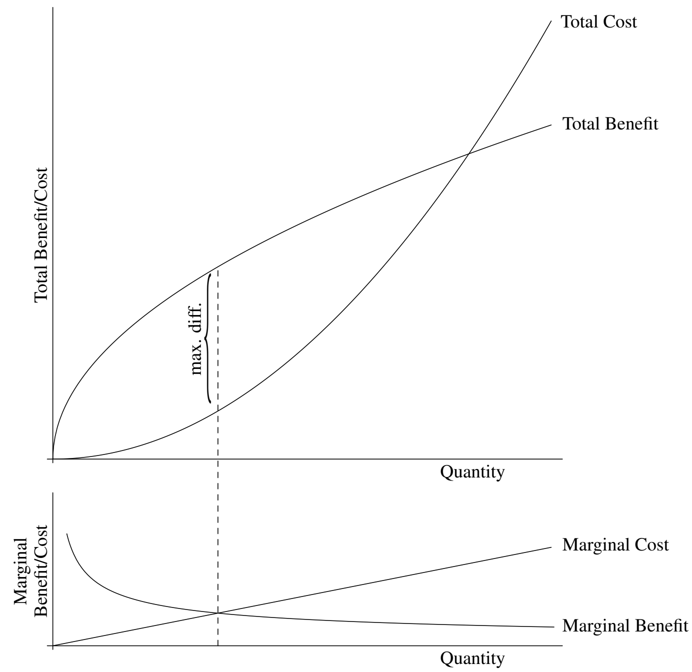


Figure 1: Maximizing Net Social Benefit

to own 10 iPods, the 11$^{\text{th}}$ one likely would not improve my well-being. As with consumers, so too with society - the marginal benefit from extended consumption tends to decrease. Indeed, it can even become negative, such that an additional unit reduces the total benefit to society.

Maximizing the net social benefit is equivalent to maximizing the difference between total social benefit and total social cost. If we assume that total social benefit and cost can be expressed as continuous and differentiable functions, say $f(x)$ and $g(x)$, respectively, then all local minima and maxima will occur where

$$\frac{\mathrm{d}}{\mathrm{d}x}(f(x) - g(x)) = 0$$

Note that

$$\frac{\mathrm{d}}{\mathrm{d}x}(f(x) - g(x)) = \frac{\mathrm{d}}{\mathrm{d}x}f(x) - \frac{\mathrm{d}}{\mathrm{d}x}g(x)$$

= Marginal Benefit − Marginal Cost. Therefore, all local minima and maxima of the total net social benefit will occur where marginal benefit is equal to marginal cost.

Figure 1 illustrates the point. The bottom portion of the figure shows marginal benefit and marginal cost - where they are equal, the difference between total social benefit and total social cost in the top portion of the figure is greatest. Marginal benefit will equal marginal costs where the total social benefit is minimized as well, and if there are multiple local maxima, marginal benefit and marginal cost will be equated at each. However, we shall restrict our explication to examples in which there exists only one global maximum of total social benefit. In such instances, maximizing total net social benefit is equivalent to equating marginal social benefit with marginal social costs.

If production is below the point that equilibrates marginal costs and marginal benefits, then the benefit to society of producing additional goods outweighs the cost of producing those extra goods. Conversely, if production is beyond the point that equilibrates marginal benefit and marginal costs, then the last good costs society more than it benefits society. Perfectly functioning free markets tend towards production levels at which marginal benefits equal marginal costs, assuming there are no externalities.

1.2 Externalities

When there exist negative externalities in production, a difference arises between the marginal social costs, and marginal private costs. If the production of each unit of some good creates a pollutant that affects people or natural resources, then a cost is borne by society for the production of each unit. However, the producer is not typically responsible for these costs, and so they are not incorporated in the costs of production.

Figure 2 illustrates the effect of negative externalities. At every level of production,

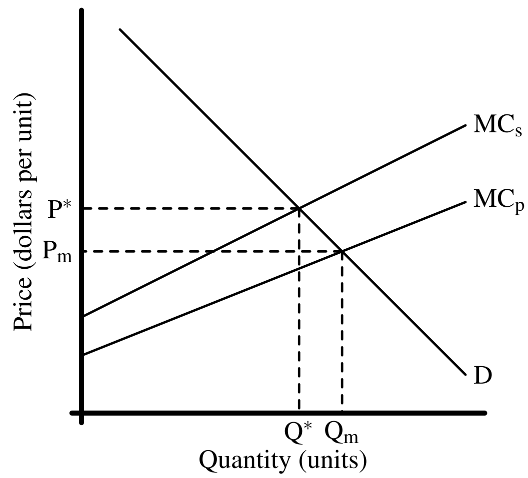


Figure 2: The effect of negative externalities

marginal social costs are greater than marginal private costs. The market is conducive to production where marginal private costs equal marginal private benefits, i.e., it is not sensitive to costs and benefits that aren't internalized. The consequent level of production occurs at a point where marginal social costs are greater than marginal social benefits: it costs society more to produce an additional unit than that extra unit benefits society. Net social benefit can be increased simply by reducing production, but the market is not conducive to such a reduction. In this case, the market has failed to promote an efficient outcome (for greater detail, see [16]).

When markets fail, governments can intervene to effect an efficient outcome. For instance, if the marginal costs due to pollution are known, a per unit tax equal to this marginal cost can be imposed. However, it's difficult to know the marginal costs of pollution at a given time. Moreover, over time the marginal costs due to pollution might increase or decrease, as technology and patterns of production change. In order for a government program to effectively bring about an efficient outcome, it must have accurate information about the marginal costs, and be flexible in response to changes in the marginal costs due to pollution. Any government program that falls short of these criteria is likely to have unintended consequences that are difficult to predict without considering the interrelatedness of the economy. For instance, over-penalizing

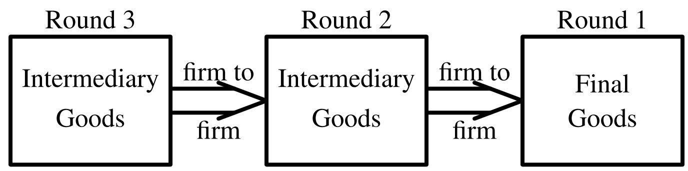


Figure 3: Flow of Goods

the steel industry will have negative effects on other industries as well.

A model that can estimate the average pollution for all sectors, the impact of government regulation, and the pollution generated not just in production of final goods, but also in production of intermediary goods, is Wassily Leontief's input-output analysis.

2 Input-Output Analysis of Pollution

2.1 Input-Output Analysis

Leontief first introduced input-output analysis in a series of journal and magazine articles, most notably in *Scientific American*. He published a compendium of these articles in 1966 [7]. The fundamental insight of input-output analysis is that the total output of a firm does not equal the total final demand for goods from that firm.

In order to produce a final product, a firm must use intermediary goods. A firm may produce its own intermediary goods, it may purchase intermediary goods from other firms, or some combination thereof. Moreover, the production of intermediary goods also requires its own supply of intermediary goods. We can continue this process recursively infinitely many times. Let us denote the production of final goods as round 1, and the production of intermediary goods for final goods as round 2. Production of round 2 intermediary goods requires its own supply of intermediary goods, which we denote round 3, etc. Figure 3 illustrates the flow of goods for the first three rounds.

Table 1: Input Output Table

from:	into:	Sector one	Sector two	Final Consumers	Total Output
Sector One		10	20	40	70
Sector Two		10	30	50	90
Labor		50	20	30	100

All intermediary goods are produced by firms; even raw materials must be mined by some firm. Thus, firms produce products that are used not just by consumers, but by other firms as well. Indeed, a firm may even consume some of its own output in furthering production. Therefore, the total output of a firm is equal to the demand for final goods from that firm *plus* the demand for intermediary goods from that firm. The aggregate exchange of goods between firms and to final consumers can be illustrated using a table, as in Table 1, which assumes just two sectors and a supply of labor.

In the table, all values represent the dollar value of units transferred from the row industry to the column industry. The rows represent output and the columns represent input. Thus, sector one produces \$10 worth of units that it uses itself, \$20 worth of units that are inputs in sector two's production, and \$40 worth of units for final consumers for a total of \$70 worth of units. Sector two produces \$10 worth of units that are used by sector one, \$30 worth of units that it consumes itself, and \$50 for final consumers for a total of \$90 worth of units. The value of output of labor to households is the opportunity cost of leisure time, i.e., it is wages foregone by not supplying labor to a productive sector.

An input-output table can be represented more succinctly if we introduce some notation: let x_i represent the total output from sector i, and x_{ij} the total output from industry i used by industry j. We denote final demand for goods from industry i as y_i. We can thus express the interaction between all sectors as a system of equations, such as

$$\begin{aligned}
x_1 =& x_{11} + x_{12} + \cdots + x_{1n} + y_1 \\
x_2 =& x_{21} + x_{22} + \cdots + x_{2n} + y_2 \\
\vdots =& \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
x_n =& x_{m1} + x_{m2} + \cdots + x_{nn} + y_n
\end{aligned}$$

The first equation can be interpreted as follows: the final demand for goods from sector one is equal to the goods sector one produces for its own use, plus the goods it produces for sectors 2 through n, plus the goods it produces for consumers.

Solving for y, we obtain:

$$\begin{aligned}
y_1 =& (x_1 - x_{11}) - x_{12} - \cdots - x_{1n} \\
y_2 =& -x_{21} + (x_2 - x_{22}) - \cdots - x_{2n} \\
\vdots =& \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
y_n =& -x_{n1} - x_{n2} - \cdots (x_n - x_{nn})
\end{aligned}$$

The first equation in this system can be interpreted thusly: the demand for final goods from sector one is equal to the total production from sector one less the amount of goods from sector one used by it or any other sector

The parenthetical grouping above has advantages both in interpretation and in computation. Interpretively, it distinguishes the net production of each sector, i.e., the total production minus the amount that a sector consumes of its own output. It may be thought of simplistically is the total production that leaves the factory. Computationally, it results in a system of equations that has as many terms (if the parenthetical groups are each considered to be one term) as there are equations. Thus, if all equations are linearly independent, the coefficients will form a basis for the set of solutions to any given demand.

To calculate the average input per unit of output, we simply divide each input for a sector by the total dollar value of output for that sector. Leontief referred to this value as the input coefficient, and denoted it as

$$a_{ij} = \frac{x_{ij}}{x_j}$$

Substituting into the above system of equations, we obtain

$$\begin{aligned}
y_1 &= (1 - a_{11})x_1 - x_2 a_{12} - \cdots - x_n a_{1n} \\
y_2 &= -x_1 a_{11} + (1 - a_{22})x_2 - \cdots - x_{2n} a_{2n} \\
\vdots &= \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
y_n &= -x_1 a_{n1} - x_{n2} a_{n2} - \cdots (1 - a_{nn})x_n
\end{aligned}$$

Which can be written in matrix form as

$$(I - A)\vec{x} = \vec{y}$$

where I is the identity matrix and A is an $n \times n$ matrix whose ij^{th} entry is a_{ij}. Given any exogenously determined demand schedule, we can use this formula to analyze the consequent production levels for each sector required to meet that demand, assuming that the $(I - A)$ matrix is non-singular. Importantly, given a change in demand for goods from one sector, we can easily determine not just the necessary change in production from that sector, but also the necessary change in production of all sectors that produce intermediary goods.

The example illustrated in Table 1 represents a closed economy, i.e. it doesn't model imports and exports. To model an open economy, we can treat imports and exports as sectors, as in Table 2

Input-output analysis has proven to be a powerful tool for analyzing the complex interrelations among sectors. It has also proven to be adaptable; modifications to it have enabled analysis of a wide array of problems, including pollution.

2.2 Application of Input-Output Analysis to Pollution

Leontief himself, in a 1970 article [8], pointed out that his model could be extended to analyze pollution. His method was to include pollution as a pseudo-sector. In Table 3 we modify Table 1 using his method, assuming hypothetical levels of pollution.

Pollution generated by a sector is entered into the table as an input into production of that sector. We can therefore calculate an input coefficient for pollution by each sector using the above method. The pollution input coefficient is the amount of pollution generated by a sector per dollar unit of output of that sector. To our above system of equations we add the following:

$$p = x_1 a_{(n+1)1} + x_2 a_{(n+1)2} + \cdots + x_n a_{(n+1),n}$$

Unfortunately, we can't incorporate the new information directly into our above matrix notation. The matrix approach required that the matrix be $n \times n$, i.e., it had to be square, whereas now we have $n+1$ equations with n terms each. Leontief, aware of this problem, suggested that the matrix method be used to solve for production levels for each sector, and that these values be plugged into the above equation to determine total pollution.

We can explicitly derive values for the amount of pollution per unit generated producing final goods, producing the first round of intermediary goods required for producing final goods, and producing the second round of intermediary goods needed for those. To do so, we calculate $\vec{p}A^{\mathrm{n}}$ where n is the round of production we're

Table 2: Input Output Table

into: from:	Sector one	Sector two	Exports	Final Consumers	Total Output
Sector One	10	20	20	40	90
Sector Two	10	30	30	50	120
Labor	50	20	0	30	100
Imports	10	20	0	40	70

Table 3: Input Output Table with Pollution

into: from:	Sector one	Sector two	Final Consumers	Total Output
Sector One	10	20	40	70
Sector Two	10	30	50	90
Labor	50	20	30	100
Pollution	40	20		60

interested in and $\vec{p}$ is the vector of pollution coefficients. The total pollution generated in rounds 1 through n is

$$\sum_{k=1}^{n} \vec{p} A^k$$

We now have a formal expression for determining the total pollution that results from a given level of demand. We can use this method to assess the impact of government regulation and consumer choices. Before applying this analysis though, it's worth considering the limitations of input-output analysis

2.3 Limitations of Input-Output Analysis

As Zhao et al. note in [18], input output analysis has been criticized for three reasons: it assumes homogeneity in production, constant proportions of inputs to outputs, and it doesn't incorporate substitution. Even the most detailed input-output tables are forced to lump together disparate products into one group, in which products are treated as homogeneous. For example, the US input-output table has one category for all plastics and rubber products. However, it's plausible that production methods for various types of plastics differ so much that they generate very different quantities of pollutants per dollar of output. Yet, input-output analysis aggregates the pollution generated by all firms that fall under this broad category, and aggregates demand for all products produced by this sector.

Input-output analysis also assumes a constant proportion of inputs to outputs, no matter the level of production. If a sector doubles its output, it's assumed that it will double all its inputs. In reality, as firms ramp up production, one of three things can happen: the ratio of the change in inputs to the change in outputs might be less than one, one, or greater than one. These three scenarios correspond to increased returns to scale, constant returns to scale, and diminishing returns to scale. Input-output analysis assumes that for all sectors as output changes, inputs change in precisely the same proportion. That is, it assumes constant returns to scale across all sectors.

Moreover, input-output analysis does not allow for substitution of inputs. Firms are assumed to use the same mix of inputs over varying levels of output. The model is insensitive to increased competition for resources, which might affect relative prices and spur substitution. It also fails to recognize that firms cannot adjust capital levels quickly; over the short-run, only labor inputs can be easily varied. A sudden increase in demand is likely to cause firms to hire more labor in the short run, and more capital in the long-run: input ratios are bound to fluctuate with changes in demand.

In addition to these theoretical drawbacks, input-output analysis presents practical challenges. The construction of input-output tables requires the accumulation and organization of large amounts of data. Even in many developed countries this is difficult to do in a timely manner; to illustrate the point, the data used in this paper is from the US benchmark input-output tables of 2002, but they weren't released until 2007. Many developing countries don't have the institutional capabilities to create input-output tables.

For countries that are capable of producing input-output tables, input-output analysis is not generally suited for inter-country comparisons. As Turner et al. point out in [17], countries typically have different classification systems for industrial sectors, and thus input-output tables between countries are rarely compatible.

Finally, in applying input-output analysis to pollution, it is limited by its neglect for pollution generated beyond productive processing. The consumption of many goods, notably automobiles, generates important pollutants that input-output analysis ignores. Also, after goods are disposed they may release pollutants, but again input-output analysis neglects this. It's worth noting, however, that there have been efforts to generate life-cycle input-output tables for tracking pollution, which incorporate pollution generated during consumption, disposal and degradation, in addition to production, notably by Carnegie Mellon's EIO-LCA (see [9]). Unfortunately however, their most recent data from the US is 12 years old.

Despite these limitations, input-output analysis is unparalleled in its ability to incorporate interdependencies among economic sectors. As Baumol and Wolff note, if input-output analysis is not used, "the trade off estimates that are used in analysis and design of policy are likely to be the grossest caricatures of the true figures"[2]. With this in mind, we turn our attention to a discussion of the data used in our input-output analysis.

3 Application of Input Output Analysis

3.1 The Data

3.1.1 Toxic Release Inventory

Since the 1986 passage of the Emergency Planning and Community Right-To-Know-Act (c.f. [1],[14]), the Environmental Protection Agency (EPA) has collected data on toxic chemicals from industrial facilities as part of its Toxic Release Inventory (TRI). After the passage of the Pollution Prevention Act in 1990, [11], the EPA has included information about waste management and source reduction in the TRI. The TRI currently covers over 20,000 facilities and 195 pollutants in the US; the data are available at [5].[1]. TRI data has many strengths: it includes facility level data; it distinguishes between how toxic chemicals are released, e.g., by land, water, or

[1]For list of all pollutants, and amounts generated in 2002, see Appendix A

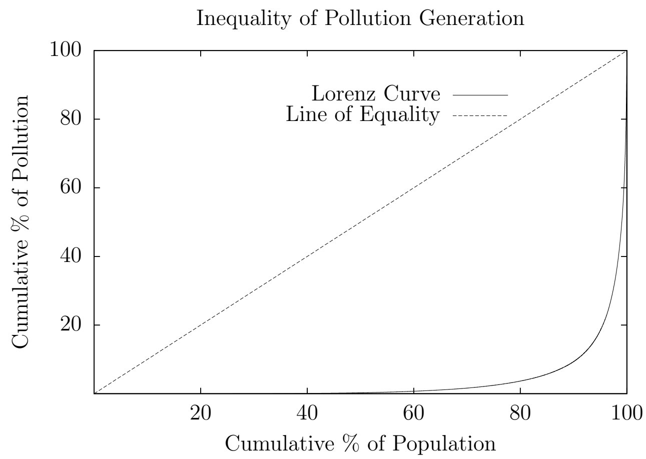


Figure 4: Inequality of Pollution Generation

disposal facilities; and it captures a large variety of toxic chemicals. These are the result of the purpose of the Toxic Release Inventory project, which is to empower citizens by informing them of exactly which chemicals are being generated by which firms, and how they're being released. Of course, TRI data has drawbacks, most importantly the fact that pollution is only measured in kilograms, which does not capture the relative consequences of different pollutants. For instance, though one kilogram of sulfuric acid may be much more dangerous than one kilogram of copper, the TRI data set, and our analysis of it, treats them as equal. For an example of an attempt to comprehensively measure the costs of different pollutants, see Mendelsohn and Muller's work, [10]. Another drawback is that firms have to meet a minimum threshold of toxic releases before being required to report them, and therefore the data don't capture facilities that release smaller amounts of pollution.

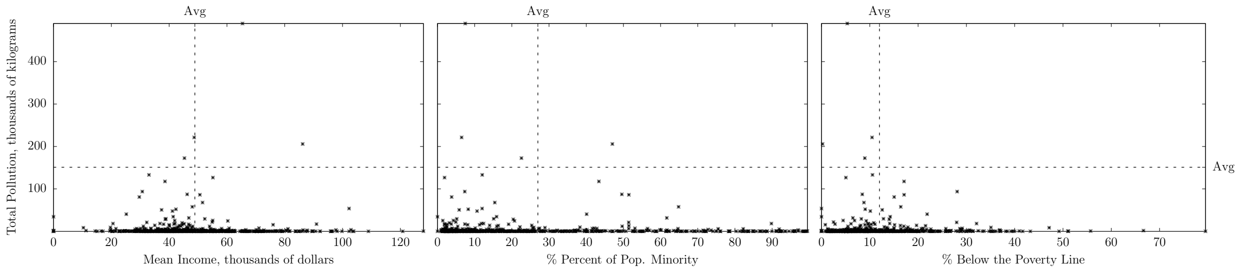
TRI data for 2002 reveal an unequal distribution of pollution generation by facilities. Indeed, the 1% of facilities that generate the most pollution account for almost 60% of all pollution captured by TRI in 2002. Such unequal distributions can be conveyed using a Lorenz curve, which is typically used to convey income or wealth inequality. in Figure 4 we've created such a curve for pollution generation. The vertical axis measures the cumulative percentage of pollution released, and the horizontal axis measures the cumulative percentage of the population. If each facility produced the same amount of pollution, then the Lorenz curve would be a straight line from the origin to the point $(100, 100)$: at every point, the percentage of the population would equal the percentage of the pollution generated. This hypothetical situation is included in the graph as the line of equality.

Below that line is the Lorenz curve representing the facilities in our data set. The large area between the line of equality and the Lorenz curve is indicative of severe inequality. Clearly, most facilities in this data set are generating only a small amount of pollution, while a handful are at the extremes.

Since the TRI data set contains facility specific information, and it includes information about each facility including its address, we can use it to test for correlations between pollution released and local socio-economic factors. There are reasons to think that more pollution would be generated in lower-income neighborhoods. An increased level of pollution might bring down property values, enabling only lower income residents to live there, for example.

The TRI data include facility addresses, including ZIP Codes. Though it's increasingly common to use ZIP Codes, or more correctly ZIP Code Tabulation Areas (ZCTA), for social research, as Krieger et al. point out in [6], this can be problematic. ZIP Codes were originally created to help facilitate mail delivery. Sometimes one ZIP Code is used for one building. The US Census Bureau created ZCTAs to facilitate their research, but they have drawbacks for social researchers. For instance, often they cover incongruous areas, and it's difficult to reliably convert from ZIP Codes to ZCTAs.

Instead then, we use the address for each firm to discover its Census Tract number. A Census Tract, as defined by the US Census Bureau [13], is a "relatively permanent statistical subdivision" that is "designed to be relatively homogeneous with respect to population characteristics, economic status and living conditions" and "usually



(a) Income (b) Minorities (c) Poverty

Figure 5: Pollution Correlations

contain between 2,500 to 8,000 inhabitants." Though the TRI data set doesn't include the Census Tract for facilities, the Federal Financial Institutions Examination Council's Geocoding System allows users to enter a single address and receive the Census Tract number and related information, see [4]. Using a script, we were able to automate the process of extracting this information for a randomly generated sample of 1,400 facilities.

Using these data, we tested the correlation between pollution released in each sample Census Tract and the median income, percent of the population that is a minority race, and the percent of the population below the poverty level. Surprisingly, none of the indicators were correlated with pollution released. The correlations coefficients for pollution and income, race and poverty were, respectively, .01, -.05, -.04. A correlation coefficient of 1 or -1 would denote a perfect correlation; the closer the value is to zero, the weaker the correlation. Our data suggest a very weak correlation. For a graphical representation of the results, see Figure 5. Note that in the figure, most data points fall below the average pollution line. The average is pulled up by the few firms that are releasing a lot of pollution.

3.1.2 BEA Input-Output Tables

The Bureau of Economic Analysis doesn't produce Leontief input-output tables. They produce five tables: a make table, a use table, a direct requirements table and two requirements tables (c.f. [12]). These tables require significant modification before being suitable for input-output analysis. We follow the method outlined in [15] to generate input-output table suitable for analysis.

Following Leontief's method, we calculate input coefficients for each of 195 pollutants listed in TRI. The result is $\vec{p_i}$ for $i = 1, 2, \ldots, 195$, where i indexes the types of pollutants. We let $\vec{p_a}$ denote the vector of total pollution, i.e., $\sum_{i=1}^{195} \vec{p_i}$. We calculate the total pollution generated in production of final goods, in production of final goods plus direct inputs of intermediary products, and in production of final goods plus direct intermediary products plus indirect intermediary products used to produce direct intermediary products. We call these rounds one, two and three, respectively (see 3). The total pollution generated in round k is

$$\sum_{n=1}^{k} \vec{p_a}\, A^n$$

3.2 Results

We find that the sector that generated the most pollution in 2002, at 1,687 kg, is primary nonferrous metal products. When production of intermediary goods through round three is incorporated, this sector generates 2,528 kg of pollution. The most significant contribution was in the form of copper and its compounds, at 864 kg in round one and 1,254 in round three. This sector uses an unusually high amount of its own output in further production: $0.32 of every dollar's worth of output. See Table 3.2 for the thirty sectors with the highest pollution through round three[2], in order from greatest to least in round three.

Note that there are several industries that would be lower in the list if it were ranked by pollution generated just in round one. For instance, the resins, rubber and artificial fibers sector is seventh on the list if we account for production through round

[2]For the full list, see Appendix B

Table 4: Thirty Sectors with Highest Pollution through Round 3

Sector	Round 1	Round 2	Round 3
Primary nonferrous metal products	1687.34	2298.48	2528.49
Boilers, tanks, and shipping containers	1051.11	1455.72	1618.74
Other electrical equipment and components	797.67	1099.17	1220.43
Foundry products	603.63	839.52	935.65
Forgings and stampings	564.97	813.67	922.51
Beverage products	501.31	748.12	859.67
Resins, rubber, and artificial fibers	407.83	630.32	735.42
HVAC and commercial refrigeration equipment	413.6	632.14	731.1
Converted paper products	507	650.94	715.08
Architectural and structural metal products	409.89	617.19	710.19
Paints, coatings, and adhesives	416.73	607.39	697.23
Motor vehicle bodies, trailers, and parts	365.62	584.31	691
Petroleum and coal products	512.73	627	680.91
Basic chemicals	392.69	589.25	679.01
Primary ferrous metal products	396.52	588.19	676.55
Electrical equipment	385.23	573.98	658.13
Household appliances	361.88	561	652.07
Pipeline transportation	445.89	584.32	638.85
Yarn, fabrics, and other textile mill products	357.8	537.22	630.55
Plastics and rubber products	372.73	535.29	615.92
Nonapparel textile products	327.69	513.57	609.83
Cutlery and handtools	358.69	526.47	598.93
Electric lighting equipment	345.94	520.29	596.18
Other fabricated metal products	347.56	517.77	593.73
Other chemical products	337.7	500.8	577.62
Turbine and power transmission equipment	307.35	477.96	560.41
Ordnance and accessories	354.34	497.21	559.52
Other general purpose machinery	293.92	472.01	556.5
Motor vehicles	176.52	410.68	545.57
Agriculture, construction, and mining machinery	255.78	442.61	536.61

three, but it would fall to thirteenth if we only considered round one. Conversely, petroleum and coal products would be higher on a list ranked by pollution in round one.

The most dramatic example of a change in ranking that results from considering production through round three is that of motor vehicles. This sector is ranked 48$^{\text{th}}$ in pollution generated in production of final goods, at 176 kg. However, if intermediary goods are considered through round three, this sector moves up 19 spots to 29$^{\text{th}}$, and accounts for 545 kg of pollution. Similarly large jumps in ranking are made by food products, which moves from 69$^{\text{th}}$ to 51$^{\text{st}}$; apparel, which moves from 81$^{\text{st}}$ to 67$^{\text{th}}$; and forestry and logging, which moves from 109$^{\text{th}}$ to 92$^{\text{nd}}$.

3.3 Policy Implications

In crafting policies to address externalities, economists tend to prefer taxes to subsides. A review of the Economic Report to the President, submitted under President Bush's administration [3], suggests politicians favor subsidies. In Chapter 3, entitled Energy and the Environment, it lists several initiatives, including the Energy Policy Act (EPACT), Energy Independence and Security Act, and Renewable Energy Production Tax Credits. Each of these calls for subsidies or tax credits (which are in effect subsidies) for cleaner technologies, and some mandate environmental standards.

The ideas upon which input-output analysis are built can help explain why politicians should favor taxes. The key insight of input-output analysis is that firms produce goods for other firms, for themselves, and for consumers, and conversely that firms must buy intermediary goods from other firms. A tax on goods whose production creates significant pollution would force substitution away from that good throughout the economy, by consumers and firms alike. A subsidy does not have this effect. Moreover, the production of the subsidized good requires intermediary goods. The increased production spurred by the subsidy may cause more pollution in the production of intermediary goods. This effect could more than offset any benefits from increased use of subsidized goods. This is precisely the conclusion reached by Baumol and Wolff [2], in their analysis of subsidies for cleaner energy production.

Of course, it's easier politically to call for a subsidy than it is to call for a tax. Input-output analysis can also inform more politically feasible policies. For instance, input-output analysis can be used to analyze the effect of cap-and-trade systems. In such systems, firms are issued permits that allow them to generate a specific amount of pollution. Firms may sell these permits to each other, so there's an incentive to pollute less and profit from selling the permit, up to the point where reducing pollution becomes more costly than the perceived gain. Input-output analysis can be used to determine precisely which industries will be most effected by a cap and trade system.

Using TRI data and the above analysis, we can determine which sectors could be taxed in order to have the greatest effect of reducing pollution. For instance, the motor vehicles industry generates less than other sectors when only final production is considered, but it accounts for a lot more when the production of its intermediary goods is included. Thus, a reduction in motor vehicle sales, caused by a tax for instance, would have a significant effect on pollution. This conclusion is obfuscated without input-output analysis.

4 Conclusion

In this paper, we analyzed pollution data for more than 20,000 facilities in the US for the year 2002. We found that a small number of firms accounts for most of the pollution generated, and that, surprisingly, pollution generation is not correlated with socio-economic indicators. Using input-output analysis, we found that the sector that generates the most pollution is primary nonferrous metal products. Several sectors changed ranking in total pollution generated when intermediary goods were included. Most notably, the motor vehicles sector was 48$^{\text{th}}$ on the list when only production of final goods was considered, but 29$^{\text{th}}$ when production of intermediary goods was considered.

This paper did not consider the effects of pollution in consumption, which can be significant. Though there are efforts to do so, notably by Carnegie Mellon's EIO-LCA, their analysis is quite dated. Future research in this area should address this shortcoming. This paper also did not incorporate a measure of the relative effects of different types of pollution. By concentrating on kilograms of pollution, rather then on costs of health consequences, this approach does allow for ranking of pollutions by severity of consequences. This is a serious drawback, and future analysis should address it. This could be done by, for instance, incorporating the work of Mendelsohn and Muller [10], though this beyond the scope of this paper.

Input-output analysis has drawbacks, including assumptions that don't reflect economic realities, and a dependence on sophisticated bureaucratic system to produce the necessary tables. However, the fundamental insight of input-output analysis, that firms produce and consume intermediary goods, as well as producing final goods, can help guide policy. This recognition reinforces the idea that taxes are more efficient than subsidies, by demonstrating inter-firm production patterns. Moreover, by directing taxes at industries that account for significant pollution not just in production of final goods, but in consumption of intermediary goods, taxes can be made to be more effective.

Appendices

A Total Pollution, by Sector

Pollutant	Kilograms released, 2002
1-Chloro-1,1-difluoroethane (HCFC-142b)	2,713,519
1,1-Dichloro-1-fluoroethane (HCFC-141b)	3,689,942
1,1,1,2-Tetrachloroethane	54,640
1,1,2-Trichloroethane	2,132,149
1,1,2,2-Tetrachloroethane	103,356
1,2-Butylene oxide	167,793
1,2-Dichlorobenzene	903,361
1,2-Dichloroethane	2,560,138
1,2-Dichloropropane	1,024,444
1,2,4-Trichlorobenzene	102,250
1,2,4-Trimethylbenzene	7,075,947
1,3-Butadiene	7,642,032
1,4-Dichlorobenzene	142,193
1,4-Dioxane	1,109,029
2-Ethoxyethanol	171,080
2-Mercaptobenzothiazole	234,407
2-Methoxyethanol	511,256
2-Methylpyridine	40,696
2-Nitropropane	42,822
2-Phenylphenol	4,542
2,4-Diaminotoluene	11,365
2,4-Dichlorophenol	6,204
2,4-Dinitrotoluene	2,591
2,6-Dinitrotoluene	1,224
3-Chloro-2-methyl-1-propene	3,655
3,3'-Dichlorobenzidine dihydrochloride	9,078
4-Nitrophenol	1,562
4,4'-Isopropylidenediphenol	2,009,328
4,4'-Methylenebis(2-chloroaniline)	7,420
4,4'-Methylenedianiline	64,859
4,6-Dinitro-o-cresol	7,694
Acetaldehyde	7,896,710
Acetonitrile	14,994,048
Acetophenone	2,913,354
Acrolein	313,680
Acrylamide	3,984,703
Acrylic acid	9,177,435
Acrylonitrile	5,753,676
Allyl alcohol	1,113,137
Allyl chloride	220,368
Aluminum (fume or dust)	18,802,372
Aluminum oxide (fibrous forms)	1,774,043
Aniline	2,078,619
Anthracene	188,874
Antimony (and its compounds)	5,876,484

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Pollutant	Kilograms released, 2002
Asbestos (friable)	3,018,866
Benzene	6,805,165
Benzoyl chloride	95,982
Benzoyl peroxide	30,444
Benzyl chloride	235,208
Biphenyl	929,252
Boron trifluoride	10,836
Bromine	252,022
Bromochlorodifluoromethane (Halon 1211)	493
Bromomethane	235,861
Bromotrifluoromethane (Halon 1301)	5,780
Butyl acrylate	457,281
Butyraldehyde	699,961
C.I. Basic Red 1	0
C.I. Direct Blue 218	2,829
C.I. Disperse Yellow 3	0
C.I. Food Red 15	0
Calcium cyanamide	40
Carbon disulfide	13,743,311
Carbon tetrachloride	669,674
Catechol	74,650
Chlorendic acid	198
Chlorine	8,175,487
Chlorine dioxide	261,270
Chloroacetic acid	4,821
Chlorobenzene	1,879,962
Chlorodifluoromethane (HCFC-22)	4,396,051
Chloroethane	619,566
Chloroform	1,811,249
Chloromethane	842,058
Chlorotetrafluoroethane (HCFC-124 and isomers)	335,762
Chlorotrifluoromethane (CFC-13)	25,216
Chromium (and its compounds)	77,087,195
Cobalt (and its compounds)	6,078,513
Copper (and its compounds)	337,227,302
Cresols	3,459,896
Crotonaldehyde	2,077
Cumene	1,356,201
Cumene hydroperoxide	182,020
Cyanides	3,048,172
Cyclohexane	3,688,039
Cyclohexanol	1,651,228
Decabromodiphenyl oxide	744,953
Di(2-ethylhexyl) phthalate	5,779,786
Dibutyl phthalate	236,799
Dichlorodifluoromethane (CFC-12)	215,282
Dichloromethane	19,504,963
Dichlorotetrafluoroethane (CFC-114)	364,362
Dichlorotrifluoroethane (HCFC-123 and isomers)	90,497
Dicyclopentadiene	449,240
Diethanolamine	1,701,279
Diethyl sulfate	2,340,763
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Pollutant	Kilograms released, 2002
Dimethyl phthalate	256,058
Dimethyl sulfate	229,892
Dimethylamine	615,534
Dinitrotoluene (mixed isomers)	573,672
Diphenylamine	733,106
Epichlorohydrin	643,210
Ethyl acrylate	778,591
Ethyl chloroformate	32,490
Ethylbenzene	11,598,869
Ethylene	20,384,968
Ethylene glycol	47,452,863
Ethylene oxide	253,490
Ethylene thiourea	4,937
Fluorine	72,259
Formaldehyde	11,560,441
Formic acid	4,881,893
Hexachlorocyclopentadiene	22,326
Hexachloroethane	63,293
Hexachlorophene	323
Hydrazine	32,281
Hydrochloric acid	272,120,206
Hydrogen cyanide	1,063,588
Hydrogen fluoride	35,573,491
Hydroquinone	302,402
Iron pentacarbonyl	18,147
Isobutyraldehyde	458,782
Isosafrole	235
Lead (and its compounds)	162,035,732
Lithium carbonate	146,916
Maleic anhydride	750,123
Manganese (and its compounds)	136,280,497
Mercury (and its compounds)	338,196
Methanol	210,593,968
Methyl acrylate	410,545
Methyl iodide	40,129
Methyl isobutyl ketone	16,155,865
Methyl methacrylate	3,048,855
Methyl tert-butyl ether	5,140,206
Molybdenum trioxide	2,110,801
Monochloropentafluoroethane (CFC-115)	181,900
n-Butyl alcohol	16,816,854
n-Hexane	32,087,333
N-Methyl-2-pyrrolidone	12,600,039
N-Methylolacrylamide	23,212
N-Nitrosodiphenylamine	58,512
N,N-Dimethylaniline	22,893
N,N-Dimethylformamide	7,959,930
Naphthalene	12,187,762
Nickel (and its compounds)	65,053,910
Nitric acid and nitrate compounds	211,592,019
Nitrilotriacetic acid	13,530
Nitrobenzene	551,888

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Pollutant	Kilograms released, 2002
Nitroglycerin	59,854
p-Nitroaniline	54,514
p-Phenylenediamine	25,027
Paraldehyde	318
Pentachloroethane	29,023
Peracetic acid	31,064
Phenol	10,361,496
Phosgene	8,101
Phosphorus (yellow or white)	241,436
Phthalic anhydride	1,558,821
Polychlorinated alkanes (C10 to C13)	277,089
Potassium bromate	113
Propargyl alcohol	65,509
Propionaldehyde	557,641
Propylene	8,817,088
Propylene oxide	1,469,654
Pyridine	1,143,042
Quinoline	31,934
Quinone	185,230
Safrole	560
sec-Butyl alcohol	1,178,129
Selenium (and its compounds)	1,467,148
Silver (and its compounds)	1,194,063
Sodium nitrite	3,986,099
Styrene	31,007,557
Styrene oxide	2
Sulfuric acid	59,948,314
tert-Butyl alcohol	5,575,329
Tetrachloroethylene	5,883,654
Tetracycline hydrochloride	2,884
Thiourea	16,985
Thorium dioxide	0
Titanium tetrachloride	155,583
Toluene	118,242,287
Toluene-2,4-diisocyanate	19,664
Toluene-2,6-diisocyanate	1,405
Toluenediisocyanate (mixed isomers)	533,176
Trichloroethylene	6,068,338
Trichlorofluoromethane (CFC-11)	304,538
Triethylamine	1,920,169
Vanadium (and its compounds)	25,396,123
Vinyl acetate	7,970,607
Vinyl chloride	3,344,701
Vinylidene chloride	138,233
Xylenes	103,955,485
Zinc (and its compounds)	342,848,627

B Pollution Rounds 1 through 3

Sector	Round 1	Round 2	Round 3
Primary nonferrous metal products	1687.34	2298.48	2528.49
Boilers, tanks, and shipping containers	1051.11	1455.72	1618.74
Other electrical equipment and components	797.67	1099.17	1220.43
Foundry products	603.63	839.52	935.65
Forgings and stampings	564.97	813.67	922.51
Beverage products	501.31	748.12	859.67
Resins, rubber, and artificial fibers	407.83	630.32	735.42
HVAC and commercial refrigeration equipment	413.6	632.14	731.1
Converted paper products	507	650.94	715.08
Architectural and structural metal products	409.89	617.19	710.19
Paints, coatings, and adhesives	416.73	607.39	697.23
Motor vehicle bodies, trailers, and parts	365.62	584.31	691
Petroleum and coal products	512.73	627	680.91
Basic chemicals	392.69	589.25	679.01
Primary ferrous metal products	396.52	588.19	676.55
Electrical equipment	385.23	573.98	658.13
Household appliances	361.88	561	652.07
Pipeline transportation	445.89	584.32	638.85
Yarn, fabrics, and other textile mill products	357.8	537.22	630.55
Plastics and rubber products	372.73	535.29	615.92
Nonapparel textile products	327.69	513.57	609.83
Cutlery and handtools	358.69	526.47	598.93
Electric lighting equipment	345.94	520.29	596.18
Other fabricated metal products	347.56	517.77	593.73
Other chemical products	337.7	500.8	577.62
Turbine and power transmission equipment	307.35	477.96	560.41
Ordnance and accessories	354.34	497.21	559.52
Other general purpose machinery	293.92	472.01	556.5
Motor vehicles	176.52	410.68	545.57
Agriculture, construction, and mining machinery	255.78	442.61	536.61
Other miscellaneous manufactured products	297.68	460.15	534.81
Other transportation equipment	253.78	425	510.22
Commercial and service industry machinery	267.49	426.98	502.53
Agricultural chemicals	266.99	404.53	470.84
Industrial machinery	227.14	382.6	460.18
Semiconductors and electronic components	251.29	389.79	456.56
Metalworking machinery	237.74	368.38	429.49
Waste management and remediation services	322.86	395.36	424.63
Air transportation	290.63	368.29	397.01
Pulp, paper, and paperboard	232.18	336.05	388
Magnetic media products	203.79	322.14	380.14
Fish and other nonfarm animals	236.9	336.44	379.94
State and local government enterprises	212.46	309.25	353.07
Crop products	207.32	305.33	348.46
Furniture and related products	166.26	287.31	347.81
Printed products	209.83	303.78	346.75
Leather and allied products	174.35	278.15	343.14
Electronic instruments	160.48	263.4	316.71
Natural gas distribution	172.08	265.34	310.25

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Sector	Round 1	Round 2	Round 3
Metal ores mining	202.41	273.61	308.62
Food products	115.53	233.29	305.41
Courier and messenger services	212.04	279.2	303.43
Audio, video, and communications equipment	130.04	239.52	302.86
Truck transportation	177.86	262.98	297.82
New residential construction	152.65	248.23	297.09
New nonresidential construction	160.22	250.98	295.79
Aerospace products and parts	127.96	232.34	293.86
Support activities for agriculture and forestry	173.7	249.65	291.52
Transit and ground passenger transportation	175.7	254.33	291.35
Soaps, cleaning compounds, and toiletries	136.73	238.95	289.03
Animal products	120.27	222.95	287.47
Nonmetallic mineral products	133.68	218.59	262.67
Maintenance and repair construction	151.58	226.81	262.55
Computer and peripheral equipment	108.12	203.02	261.08
Medical equipment and supplies	129.77	216.46	259.27
Mining support services	126.99	213.34	257.82
Apparel	72.63	185.43	247.76
Rail transportation	136.65	198.01	228.74
Pharmaceuticals and medicines	139.2	192.65	218.92
Wood products	93.44	167.34	216.53
Tobacco products	115.89	173.98	199.53
Automotive repair and maintenance	84.65	156.87	195.71
Water transportation	80.05	155.64	194.37
Nonmetallic mineral mining and quarrying	109.82	163.98	193.77
Coal mining	93.13	155.48	190.53
Food services and drinking places	90.32	150.25	184.67
All other administrative and support services	118.82	164.37	183.86
General state and local government services	107.46	154.09	175.97
General Federal defense government services	91.07	132.85	157.64
Newspapers, books, and directories	67.4	123.31	150.89
Internet publishing and broadcasting	52.28	114.89	148.91
Scenic and sightseeing transportation and support activities	69.9	121.53	144.65
Electronic, commercial, and household goods repair	72.69	118.24	142.37
Oil and gas extraction	60.39	107.95	133.27
Warehousing and storage	68.61	108.86	128.32
Educational services	62.87	103.44	125.27
Electric power generation, transmission, and distribution	65.36	104.5	123.88
Accommodation	73.53	104.51	121.06
Civic, social, professional and similar organizations	56.74	95.83	118.28
Hospital care	57.73	96.66	116.5
Amusements, gambling, and recreation	65.29	98.49	116.02
Forestry and logging products	23.35	70.67	107.9
Scientific research and development services	42.99	81.9	103.09
Social assistance	43.78	81.84	102.08
Nursing and residential care	54.63	85.24	101.56
General Federal nondefense government services	44.57	81.59	101.31
Retail trade	45.43	80.98	100.26
Personal and laundry services	43.6	79.51	98.84
Telecommunications	38.1	70.56	90.28
Real estate	63.59	80.14	87.29
Federal Government enterprises	42.58	72.38	87.16

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Sector	Round 1	Round 2	Round 3
Ambulatory health care services	42.71	71.24	86.2
Cable networks and program distribution	26.48	61.58	85.26
Wholesale trade	36.88	68.25	84.43
Machinery and equipment rental and leasing	29.14	61.85	81.36
Other professional and technical services	38.42	66.11	80.64
Architectural, engineering, and related services	29.34	58.16	73.89
Automotive equipment rental and leasing	21.77	52.81	70.9
Water, sewage and other systems	24.54	53.83	69.89
Consumer goods and general rentals	30.18	52.31	66.77
Religious, grantmaking, and social advocacy	16.89	48.43	65.91
Travel arrangement and reservation services	20.29	49.29	65.56
Specialized design services	19.87	48.47	63.49
Data processing services	19.38	45.31	60.08
Other information services	21.47	44.21	57.56
Advertising and related services	17.58	42.83	56.96
Motion pictures and sound recordings	15.66	38.87	53.88
Performing arts, spectator sports, and museums	19.5	40.18	52.99
Management of companies and enterprises	19.15	39.02	52.43
Computer systems design and related services	15.66	38.27	50.93
Owner-occupied dwellings	15.86	36.64	48.03
Radio and television broadcasting	13.05	30.5	45.51
Software publishers	9.91	31.29	43.56
Management and technical consulting services	11.21	31.51	42.87
Rights to nonfinancial intangible assets	22.74	34	39.79
Accounting, tax preparation, bookkeeping, and payroll services	13.19	29.46	38.29
Legal services	9.82	27.33	36.25
Securities, commodity contracts, investments, and related activities	7.21	21.14	30.75
Employment services	9.08	19.71	25.67
Insurance carriers and related services	5.78	16.38	24.57
Monetary authorities, credit intermediation and related activities	4.44	15.86	23.91
Funds, trusts, and other financial vehicles	1.32	10.36	22.6
Private household services	0	0	0

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